MA114 Worksheet for Section 8.1

Concepts

1. A ball is hurled into the air from the ground. Its height, velocity and acceleration are all functions of time, $h(t)$, $v(t)$, and $a(t)$ respectively. Suppose we know $v(t) = -32t + 200$ feet per second.

   (a) Explain why the *displacement* of the ball between time t_1 and time t_2 is given by $\int_{t_1}^{t_2} v(t)\,dt$.

   (b) Explain why the *total distance* the ball travels from time t_1 and time t_2 is given by $\int_{t_1}^{t_2} |v(t)|\,dt$. It may help to ask yourself first what the difference between displacement and total distance is.

   (c) When does the ball hit the ground? When does it reach its maximum height?

   (d) What is the displacement of the ball from $t_1 = 1$ second to $t_2 = 1.5$ seconds?

   (e) What is the total distance the ball travels during that time?

2. Let $C(x)$ be the cost of producing x items. We call $C'(x)$ the *marginal cost.*

   (a) Explain why $C'(x)$ is the approximate cost to produce the x^{th} item. (See section 4.7).

   (b) If the marginal cost of manufacturing x units of product is $C'(x) = 0.006x^2 - 1.5x + 8$ (measured in dollars per unit) and the start-up cost is $C(0) = 1,500,000$ find the cost of producing 2000 units.

3. Since you're busy looking up section 4.7 anyway, while you are there look up *marginal revenue.* If the marginal revenue from selling x items is $90 - 0.2x$ and the revenue generated from selling the first 100 items is 8800 dollars, what is the revenue from the sale of the first 200 items?

4. Consider the integral $I = \int_0^1 \sqrt{1 + 4x^2}\,dx$.

   (a) Calculate I by antidifferentiation. You will need to use the integral table provided with chapter 7. Round your answer to 6 decimal places.

   (b) I can be thought of as the area under some curve. Decide which curve and draw the region in question. Does the area look about "right," numerically?

   (c) I can also be thought of as the length of a graph $y = f(x)$, from $x = 0$ to $x = 1$. Find and plot such a function. Does your graph appear to have about the "right" length?

   (d) Is more than one function f possible in the previous part? If so, find and plot another possible function.

   (e) Use the above integral to explain the following inequality.

   $$\sqrt{2} < \frac{\sqrt{5}}{2} + \frac{\ln(2 + \sqrt{5})}{4}$$

5. The linear density of a rod of length 4 meters is given by $\rho(x) = 9 + 2\sqrt{x}$, measured in kilograms per meter, where x is measured in meters from one end of the rod. Find the total mass of the rod.

6. Breathing is cyclic and a full respitory cycle from beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 Liters per second. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow in the lungs. Use this model to find the volume of inhaled air in the lungs at time t.