MA114 Worksheet for Section 8.2

Skills

1. Sketch the region in the first quadrant bounded by the given curves and find the volume of the solid formed when this region is revolved around the $x-$axis.
   (a) $y = x^4, y = 1, x = 0.$
   (b) $y = x^2, y = x^3.$

2. Sketch the region bounded by the given curves and find the volume of the solid of revolution when the region is revolved around the $y-$axis.
   (a) $y = \sqrt{x}, y = x^2.$
   (b) $y = x^2, y = 2^x, x = 0.$

3. Sketch the region bounded by the given curves and find the volume of the solid of revolution formed when the region is revolved around the line $y = a$.
   (a) $x = y^2 - 4, y = 2 - x, a = 2.$
   (b) $y = \sqrt{x}, y = 0, x = 1, a = 1.$

4. Compute the following volumes using either the method of washers or the method of shells.
   (a) The volume of the solid that results from rotating the region bounded by $y = x$ and $y = x^2$ around the $x-$axis.
   (b) The volume of the solid that results from rotating the region bounded by $x + y = 1, x = 0$, and $y = 0$ about the x-axis.
   (c) The volume of the solid that results from rotating the region bounded by $y = -\frac{1}{x}, y = 0, x = 1$, and $x = 3$ about the $x-$axis.

Concepts

1. Find the formula for the volume of a cone with radius r and height h.

2. Find the formula for the volume of a pyramid with height h and a square base with sides of length l.

3. Find the formula for the volume of a sphere of radius r.

4. Find the volume of a pyramid with height h and base an equilateral triangle with side a.

5. A hole of radius r is bored through the center of a sphere of radius $R > r$. Find the volume of the remaining portion of the sphere.

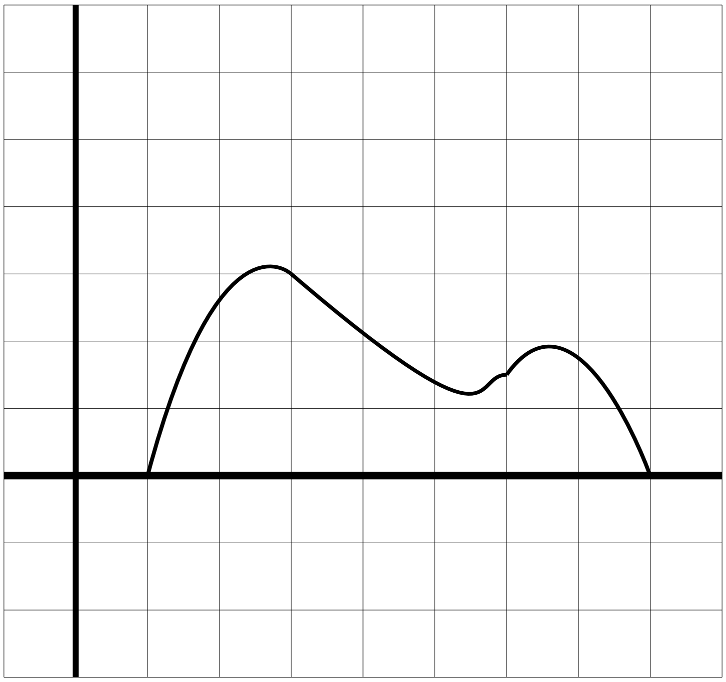
6. Suppose you have a cylindrical shaped fuel tank of radius 4 feet and length (height) 25 feet. A fuel distributor will fill "a quarter" of your tank for a certain price. Which do you do?
   (a) Stand the tank upright so that the flat sides are parallel to the ground and have him fill your tank up 25/4 feet (which is "one quarter" of the height).
   (b) Set the tank on its side so that the flat sides are perpendicular to the ground and have him fill your tank up 1 foot (which is "one quarter" of the new height).

   Believe it or not, it was a dispute similar to this that led to the creation of integral calculus by Leibniz, except instead of fuel tanks he was interested in kegs of wine. A wine keg has a slightly more complicated shape since it bulges near the center, but the problem is basically the same. Wine was typically sold by the quarter keg, but different sellers measured "a quarter" in the two ways described above, and Leibniz wanted to know how to get the most wine for his money!

7. A hemispherical bowl of radius R contains water to a depth h. Find the volume of the water in the bowl.

8. Suppose a wedge is cut from a solid right circular cylinder (like a wedge cut from a tree by an axe) such that one side of the wedge is horizontal and the other is inclined at 30°. Assuming the wedged sides meet in a diameter of the circular cross section and the diameter of the cylinder is 10 meters, find the volume of the wedge removed. Hint: The vertical cross sections of the wedge are right triangles.

9. If the region shown in the figure below is rotated about the y-axis to form a solid, use the Midpoint rule with $n = 7$ to estimate the volume of the solid. Note that the grid is comprised of unit squares.



Maple

1. Our goal is to find the volume obtained by rotating the region under the curve $y = \tan x, 0 \leq x \leq \pi/4$, about the y-axis.

   (a) Set up the volume integral using your choice of methods. See if Maple can evaluate the integral. If it can't, try setting the problem up a different way.

   (b) Choose one of your integrals and approximate it using the midpoint method. Start with $n = 10$ and use larger and larger values until you are convinced you have 4 decimal place accuracy.