MA114 Worksheet for Section 8.3

Skills

Find the arc length of the given curve:

1. $y = \frac{1}{3}(x^2+2)^{\frac{3}{2}}, 0 \le x \le 1$.
2. $y = \dfrac{x^4}{4} + \dfrac{1}{8x^2}, 1 \le x \le 3$.
3. $y = \dfrac{x^2}{2} - \dfrac{\ln x}{4}, 2 \le x \le 4$.
4. $y = \ln(\cos x), 0 \le x \le \pi/4$.
5. $y = \ln(1-x^2), 0 \le x \le \frac{1}{2}$.
6. $y = e^x, 0 \le x \le 1$.
7. $y = \ln x, 1 \le x \le \sqrt{3}$.
8. $x^2 = 64y^3$ from $(8,1)$ to $(64,4)$.

Concepts

1. A bomb is dropped from an aircraft flying at 200 m/s at an altitude of 4500 m . The parabolic trajectory of the bomb is described by the equation
$$y = 4500 - \frac{x^2}{8000}$$
until it hits the ground, where y is its height in meters above the ground and x is the horizontal distance traveled in meters. Calculate the distance traveled by the bomb from the time it is dropped to the time it hits the ground. Express your answer correct to the nearest meter.
2. Find the perimeter of the area enclosed by $y^3 = x^2$ and $x + 3y - 4 = 0$.
3. Find the perimeter of the area enclosed by $y = 3(x-1)^{3/2}$ and $y = 6(x-1)$.
4. Find the length of one loop of $9y^2 = x(x-3)^2$. Suggestion: Use maple to plot this curve first.
5. Find the point P on the graph of $y = \frac{2}{3}x^{3/2}$ to the right of the y-axis so that the length of the graph $y = \frac{2}{3}x^{3/2}$ from the origin to P is $\frac{52}{3}$.

Maple

1. Use Simpson's rule with $n = 10$ to estimate the arc length of the given curve.
   (a) $y = x^3, 0 \le x \le 1$.
   (b) $y = \sin x, 0 \le x \le \pi$.
   (c) $y = e^x \cos x, 0 \le x \le \pi/2$.
2. An ellipse centered at the origin is given by the equation $\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$. Set up an integral for the arc length of the ellipse from $x = 0$ to $x = a/2$. The resulting integral is particularly notorious, and is known as an *elliptic integral (of the second kind)*. Use Simpson's rule, starting with $n = 10$ to approximate the integral. Use larger and larger values of n to get better approximations.