MA114 Worksheet for Section 9.1

Skills

1. Use integration by parts to evaluate each of the following integrals.

   (a) $\displaystyle\int xe^{-3x}\,dx$

   (b) $\displaystyle\int x\cos x\,dx$

   (c) $\displaystyle\int \sqrt{x}\ln x\,dx$

   (d) $\displaystyle\int x^{-2}\ln x\,dx$

   (e) $\displaystyle\int \arcsin x\,dx$

   (f) $\displaystyle\int \arctan x\,dx$

   (g) $\displaystyle\int (\ln x)^2\,dx$

   (h) $\displaystyle\int_0^{\pi} e^x\cos x\,dx$

   (i) $\displaystyle\int_0^1 x^2e^{-x}\,dx$

   (j) $\displaystyle\int x^2\sin x\,dx$

   (k) $\displaystyle\int x\cos^2 x\,dx$

   (l) $\displaystyle\int x^3e^{x^2}\,dx$ [Hint: $u = x^2$]

   (m) $\displaystyle\int xe^x\sin x\,dx$

   (n) $\displaystyle\int x^2(\ln x)^2\,dx$

   (o) $\displaystyle\int x\arctan x\,dx$

Concepts

1. Derive the formula $\displaystyle\int \ln(x+\sqrt{x^2+a^2})\,dx = x\ln(x+\sqrt{x^2+a^2}) - \sqrt{x^2+a^2} + C.$

2. Derive the formula $\displaystyle\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2} + C.$

3. Establish the following reduction formulas where $n > 1$ is an integer.

   (a) $\displaystyle\int x^n\arcsin x\,dx = \frac{x^n+1}{n+1}\arcsin x - \frac{1}{n+1}\int \frac{x^{n+1}}{\sqrt{1-x^2}}\,dx.$

   (b) $\displaystyle\int x^n(ax+b)^{1/2}\,dx = \frac{2x^n(ax+b)^{3/2}}{(2n+3)a} - \frac{2bn}{(2n+3)a}\int x^{n-1}(ax+b)^{1/2}\,dx.$

   (c) $\displaystyle\int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n}\int \sin^{n-2}x\,dx.$

4. Find the volume of the solid generated by revolving the area under the graph of $y = \sin x$ from $x = 0$ to $x = \pi/2$ about the y-axis.

5. Find the volume of the solid generated by revolving the area under the graph of $y = x\sqrt{\sin x}$ from $x = 0$ to $x = \pi/2$ about the x-axis.

6. A particle that moves along a straight line has velocity $v(t) = t^2 e^{-t}$ meters per second after t seconds. How far will it travel during the first t seconds?

Maple

1. Use the change of variable $x = u^3$ to express the integral $\displaystyle\int \frac{\sqrt{4 + x^{2/3}}}{\sqrt[3]{x}}\, dx$ in simpler forms. Then use Maple's *int* command to evaluate the resulting integral. Substitute back to get the result in terms of x.

2. Use an appropriate change of variable to express the integral $\displaystyle\int \frac{1}{1 - 3\sqrt{x} + 2\sqrt[3]{x}}\, dx$ in a simpler form. Then use Maple's *int* command to evaluate the resulting integral. Substitute back to get the result in terms of x. Hint: let $u = x^n$ for som integer n.