MA114 Worksheet for Section 9.3

Skills

1. Evaluate the following integrals.

   (a) $\displaystyle\int \cos^4 x\, dx$

   (b) $\displaystyle\int_0^{\pi/4} \sin^4 x \cos^2 x\, dx$

   (c) $\displaystyle\int \sin^5 2x \cos^4 2x\, dx$

   (d) $\displaystyle\int \sin^3 x\sqrt{\cos x}\, dx$

   (e) $\displaystyle\int \frac{\cos^3 x}{\sqrt{\sin x}}\, dx$

   (f) $\displaystyle\int \frac{\cos^2 \sqrt{x}}{\sqrt{x}}\, dx$

   (g) $\displaystyle\int \frac{1-\sin x}{\cos x}\, dx$

   (h) $\displaystyle\int_0^{\pi/4} \tan^2 x \sec^4 x\, dx$

   (i) $\displaystyle\int_0^{\pi/3} \tan^5 x \sec x\, dx$

   (j) $\displaystyle\int \frac{1}{\sin^4 x}\, dx$

2. Evaluate the following integrals.

   (a) $\displaystyle\int_{1/2}^{\sqrt{3}/2} \frac{1}{x\sqrt{1-x^2}}\, dx$

   (b) $\displaystyle\int_0^2 x^3\sqrt{4-x^2}\, dx$

   (c) $\displaystyle\int \frac{x}{\sqrt{1-x^2}}\, dx$

   (d) $\displaystyle\int x\sqrt{4-x^2}\, dx$

   (e) $\displaystyle\int \sqrt{x^2+1}\, dx$

   (f) $\displaystyle\int \frac{x^2}{(4-x^2)^{3/2}}\, dx$

   (g) $\displaystyle\int \frac{1}{(1+x^2)^2}\, dx$

   (h) $\displaystyle\int \frac{1}{\sqrt{x^2+4x+8}}\, dx$

   (i) $\displaystyle\int e^t\sqrt{9-e^{2t}}\, dt$

Concepts

1. Find the average value of the function $f(x) = \sin^2 x \cos^3 x$ on the interval $[-\pi, \pi]$.

2. Find the area between $y = \sin x, y = \sin^3 x, x = 0$, and $x = \pi/2$.

3. Find the area of the region bounded by the hyperbola $9x^2 - 4y^2 = 36$ and the line $x = 3$.

4. Find the volume obtained by rotating the region bounded by $y = \tan^2 x, y = 0, x = 0$, and $x = \pi/4$ about the x-axis.

5. A torus is generated by rotating the circle $x^2 + (y - R)^2 = r^2$ about the x-axis. Find the volume enclosed by the torus.

6. A particle moves on a straight line with a velocity function $v(t) = \sin \omega t \cos^2 \omega t$. Find its position function $s = f(t)$ if $f(0) = 0$.

7. Use all the techniques of integration you know to evaluate the following brutal integrals. You may have to use more than one technique for each problem.

   (a) $\displaystyle\int \ln(1 + x^2)\, dx$

   (b) $\displaystyle\int_0^1 (1 + \sqrt{x})^8\, dx$

   (c) $\displaystyle\int x \arcsin x\, dx$

   (d) $\displaystyle\int x^5 e^{-x^3}\, dx$

   (e) $\displaystyle\int_{\pi/4}^{\pi/3} \frac{\ln(\tan x)}{\sin x \cos x}\, dx$

   (f) $\displaystyle\int \frac{x^4}{x^{10} + 16}\, dx$

   (g) $\displaystyle\int \frac{1}{e^{3x} - e^x}\, dx$

   (h) $\displaystyle\int \frac{x \ln x}{\sqrt{x^2 - 1}}\, dx$