Complete ideals in two-dimensional regular local rings

by Mee-Kyoung Kim, Sungkyunkwan University

Abstract : This is joint work with William Heinzer.

Let $(R, \mathfrak{m}, k)$ be a two-dimensional regular local ring with the field of fractions $\mathcal{Q}(R)$. Let I be a complete $\mathfrak{m}$-primary ideal. Zariski's unique factorization theorem implies that $I = I_1^{n_1} \cdots I_h^{n_h}$, where the n_i are positive integers and the I_i are distinct simple complete ideals. To examine properties of the simple factors of I, it is convenient to write $I = K^n L$, where K is a simple complete $\mathfrak{m}$-primary ideal and either $L = R$, or L is a complete $\mathfrak{m}$-primary ideal that does not have K as a factor. Let V be the unique Rees valuation ring of K and let v denote the Rees valuation associated with V. Since $V \in \operatorname{Rees} I$, we also have $V = R[It]_Q \cap \mathcal{Q}(R)$ for some $Q \in \operatorname{Min}(\mathfrak{m} R[It])$. Let $J := (a, b)R$ be a reduction of I and let $\overline{(\frac{a}{b})_v}$ denote the image of $\frac{a}{b}$ in the residue field $k(v)$ of V. We assume that the field k is relatively algebraically closed in the residue field $k(v)$ of V. We will show that $\left[k(v) : k\left(\overline{(\frac{a}{b})_v}\right)\right] = n$. Next, we will describe a minimal generating set of I, by considering the subideal $K_V := \{a \in K \mid aV \neq KV\}$ of K. It allows to conclude that the quotient ring $\frac{R[It]}{Q}$ is a two dimensional normal Cohen-Macaulay standard graded domain over k that has minimal multiplicity at its maximal homogeneous ideal with this multiplicity being n.