Certain Jet Schemes have Rational Singularities

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This is a joint work with Shihoko Ishii and Akiyoshi Sannai.

Let X be an affine scheme over a field k of characteristic 0. We denote by X_m the m-jet scheme of X. We ask the following question.

Question. When does X_m have rational singularities for every $m \geq 1$?

We consider the case where $X = \mathrm{Spec}\,(k[X_1, \ldots, X_N]/(f))$, where f is a homogeneous polynomial of degree d. Then we show the following.

Theorem. Assume f is a generic homogeneous hypersurface of degree d. Then X_m has rational singularities for every $m \geq 1$ if and only if $N > d^2$.

To prove this, we use Fedder's criterion to show that the reduction modulo p of X_m is F-rational for every $m \geq 1$ and $p \gg 0$.