3.6 Variation (OR Proportion)

* We say that y varies directly as x (or we say that y is directly proportional to x) if

$$y = k \cdot x \qquad k = \text{constant}$$

* We say that y varies inversely as x (or we say that y is inversely proportional to x) if

$$y = \frac{k}{x} \qquad k = \text{constant}$$

Example: Suppose that q is inversely proportional to the sum of x and y. If $x = 0.5$ and $y = 0.7$ then $q = 1.4$.

Find a formula for q.

Ans: $q = \dfrac{k}{x+y}$

Now $1.4 = \dfrac{k}{0.5 + 0.7}$ $\quad \therefore k = 1.4 \cdot (1.2)$

$\overset{!}{=} 1.68$

Thus $\boxed{q = \dfrac{1.68}{x+y}}$

Example: The intensity of illumination I from a source of light varies inversely as the square of the distance d (in feet) from the source.

(a) Express I in terms of d and a constant of variation k

(b) A search light has an intensity of 1,000,000 candle-power at a distance of 50 feet. Find k.

(c) Approximate the intensity of the search light at a distance of 1 mile.

Ans:

(a) $$I = \frac{k}{d^2}$$

(b) $$1{,}000{,}000 = \frac{k}{(50)^2} \Rightarrow k = 2.5\,10^9$$

(c) 1 mile = 5,280 feet Thus

$$I = \frac{2.5\,10^9}{(5{,}280)^2} \simeq \underline{89.67}$$