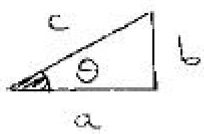
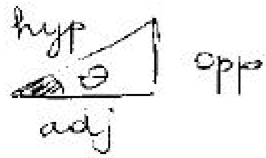
5.2 Trigonometric functions of Angles

Let θ be an acute angle and let us consider the right triangle



or



For such a θ the following 6 ratios are well defined and they are called

* $\boxed{\text{Trigonometric functions}}$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

$$\csc\theta = \frac{\text{hyp}}{\text{opp}} \qquad \sec\theta = \frac{\text{hyp}}{\text{adj}} \qquad \cot\theta = \frac{\text{adj}}{\text{opp}}$$

!! Notice that once you know $\sin\theta$ and $\cos\theta$ then you know all remaining trig. function

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta}$$

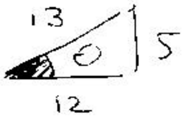
$$\tan\theta = \frac{1}{\cot\theta}$$

* $\boxed{\text{Pythagorean identities}}$: from $a^2 + b^2 = c^2$ it follows that $\left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2 = 1$ $\therefore$

$\cos^2\theta + \sin^2\theta = 1$ and also

$$1 + \tan^2\theta = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

* **Ex.** find the 6 trigonometric functions from

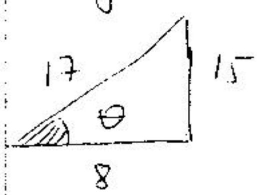


$\cos\theta = \frac{12}{13}$ $\sin\theta = \frac{5}{13}$

$\tan\theta = 5/12$ $\cot\theta = 12/5$ $\csc\theta = 13/5$

$\sec\theta = 13/12$

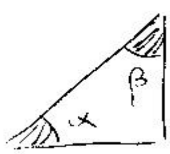
* **Ex:** Find the exact values of the 6 trigonometric functions for the acute angle θ such that $\cos\theta = 8/17$



hence $\sin\theta = 15/17$ $\tan\theta = \frac{15}{8}$

$\cot\theta = 8/15$ $\csc\theta = \frac{17}{15}$ $\sec\theta = \frac{17}{8}$

* **Notice** that if α and β are complementary angles, that is $\alpha + \beta = 90°$ then



$\sin\alpha = \cos\beta = \cos(90 - \alpha)$

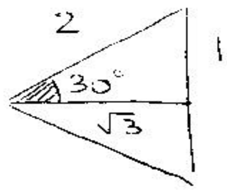
$\cos\alpha = \sin\beta = \sin(90 - \alpha)$

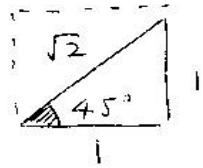
etc....

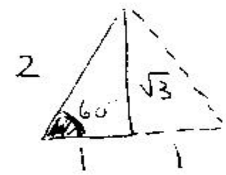
* Let' us find the trig values for the following angles

$\pi/6 = 30°$ $\pi/4 = 45°$ $\pi/3 = 60°$

$\theta = 30°$	$\theta = 45°$	$\theta = 60°$







Using these three triangles we get

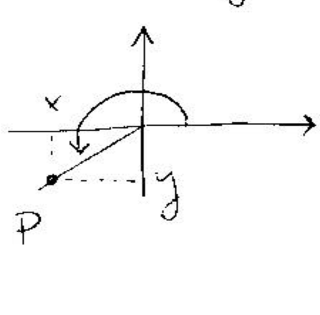
$\theta = 30°$	$\theta = 45°$	$\theta = 60°$
$\cos\theta = \sqrt{3}/2$	$\cos\theta = \frac{1}{\sqrt{2}} = \sqrt{2}/2$	$\cos\theta = 1/2$
$\sin\theta = 1/2$	$\sin\theta = \sqrt{2}/2$	$\sin\theta = \sqrt{3}/2$
$\tan\theta = \sqrt{3}/3$	$\tan\theta = 1$	$\tan\theta = \sqrt{3}$
$\cot\theta = \sqrt{3}$	$\cot\theta = 1$	$\cot\theta = \sqrt{3}/3$
$\csc\theta = 2$	$\csc\theta = \sqrt{2}$	$\csc\theta = 2/3\sqrt{3}$
$\sec\theta = \frac{2}{3}\sqrt{3}$	$\sec\theta = \sqrt{2}$	$\sec\theta = 2$

We are now ready to define the trig. functions of any angle θ.

Let $P(x,y)$ be any point different from the origin on the terminal side of θ. Let

$r = \sqrt{x^2+y^2}$

$$\sin\theta = \frac{y}{r} \qquad \cos\theta = \frac{x}{r}$$



$$\tan\theta = y/x \qquad \cot\theta = x/y$$

$$\csc\theta = \frac{r}{} \qquad \sec\theta = \frac{r}{}$$

* Verify the identity $\cos^2(2\theta) - \sin^2(2\theta) = 2\cos^2(2\theta) - 1$ by transforming the left-hand side into the right-hand side.

Recall that from $\sin^2(\alpha) + \cos^2(\alpha) = 1$ we get $\sin^2(\alpha) = 1 - \cos^2(\alpha)$. Thus

$$\cos^2(2\theta) - \sin^2(2\theta) = \cos^2(2\theta) - \left[1 - \cos^2[2\theta]\right]$$
$$\stackrel{!}{=} 2\cos^2(2\theta) - 1 \quad \checkmark$$

* $\cos^2\theta\,(\sec^2\theta - 1) \stackrel{?}{=} \sin^2\theta$

$$\cos^2\theta\,(\sec^2\theta - 1) = \cos^2\theta\left[\frac{1}{\cos^2\theta} - 1\right] = 1 - \cos^2\theta$$
$$= \sin^2\theta \quad \checkmark$$

* $\log[\tan\theta] \stackrel{?}{=} \log(\sin\theta) - \log(\cos\theta)$

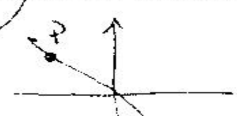
$$\log[\tan\theta] = \log\left[\frac{\sin\theta}{\cos\theta}\right] = \log\sin\theta - \log\cos\theta \quad \checkmark$$

* Find the exact values of the 6 trigonometric functions of θ if θ is in standard position and P is on the terminal side: $P(4,-3)$

$r = \sqrt{4^2 + (-3)^2} = \sqrt{25} = 5 \qquad \sin\theta = -3/5 \quad \cos\theta = 4/5$

etc...

* ... same as above ... and the terminal side of θ is in the II quadrant and on the line $y = -4x$



pick $P(-1,4)$

$r = \sqrt{17}$ etc...