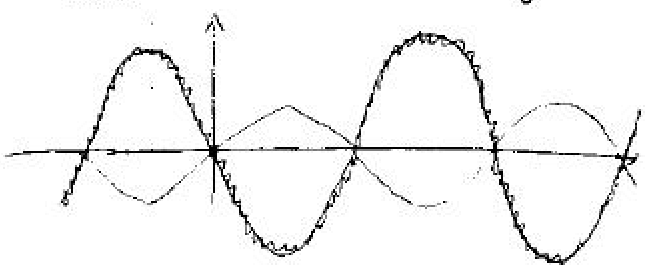
5.5 Trigonometric Graphs

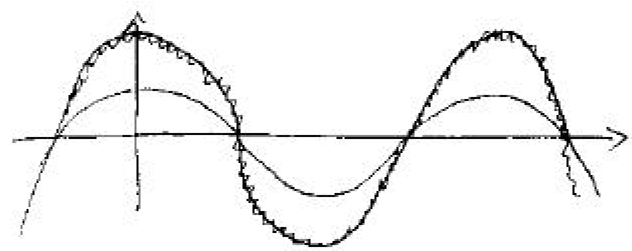
Goal: study the graphs of

$$y = a\sin(bx+c) \quad \& \quad y = a\cos(bx+c)$$

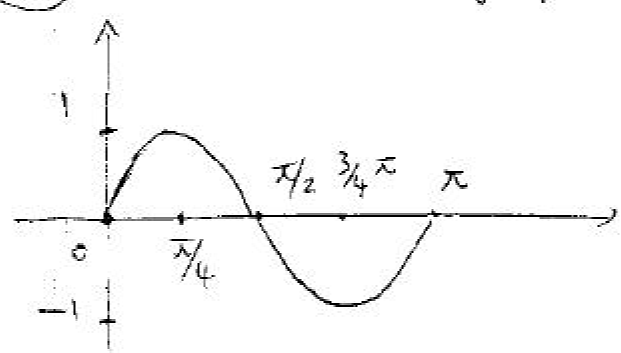
for $a, b, c \in \mathbb{R}$ and by resorting to the graphs of $y = \sin x$ and $y = \cos x$.

Ex: Sketch $y = -3\sin x$, $y = 2\cos x$





Ex: Sketch the graph of $y = \sin(2x)$ in $[0, \pi]$



notice that the graph gets compressed of a factor of 2.

i.e. the period of $\sin 2x$ is $2\pi/2 = \pi$

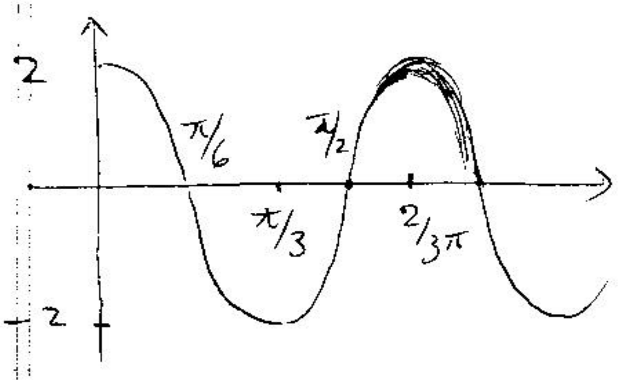
In general $y = a\sin(bx)$ and $y = a\cos(bx)$

have amplitude $= |a|$ and period $= \dfrac{2\pi}{|b|}$

Ex: Find the amplitude and the period of

$y = 2\cos(-3x)$ $\qquad |a| = 2$ amplitude

$\frac{2\pi}{|-3|} = \frac{2}{3}\pi$ period



also notice that

$y = 2\cos(-3x)$

$\stackrel{!}{=} 2\cos(3x)$

Next: consider $y = a\sin(bx) + c$

or $y = a\cos(bx) + c$

in this case there is a vertical shift of "c" units.

This is different from considering

$y = a\sin(bx + c)$ or $y = a\cos(bx + c)$

the number $-c/b$ is called phase shift

An interval containing exactly one cycle of sine or cosine can be found by solving $0 \leq bx + c \leq 2\pi$

Assuming $b>0$ we get that

$$0 \le bx + c \le 2\pi \iff -c/b \le x \le \frac{2\pi - c}{b}$$

Notice that the amplitude of this ↑ interval is $\frac{2\pi - c}{b} - (-c/b) = \frac{2\pi}{b}$

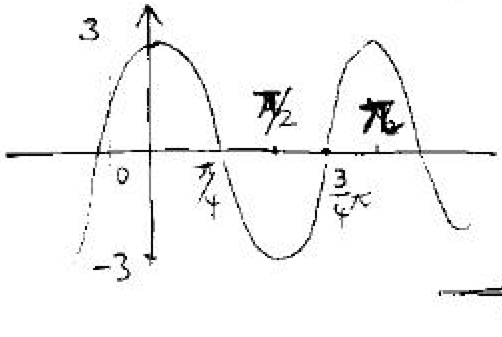
Ex: Find the amplitude, period and phase shift of $y = 4\sin\left(\frac{1}{3}x - \pi/3\right)$

amplitude $= 4$; period $= \frac{2\pi}{1/3} = 6\pi$; phase shift $= -\frac{(-\pi/3)}{1/3} = \pi$

Ex: Same for $y = \cos(2x - \pi) + 2$

$|a| = 1$; period $= \frac{2\pi}{2} = \pi$; phase shift $= \pi/2$.

Ex: The graph of an equation is shown below. (a) Find the amplitude, period and phase shift. (b) Write the equation in the form $a\sin(bx+c)$



$a = 3$; period $= \pi$

phase shift $= -\pi/4$

$\therefore 3\sin(bx+c)$ $\quad 2\pi/b = \pi$

$\Longrightarrow b = 2$; $-c/2 = -\pi/4 \Longrightarrow c = \pi/2$