#6.1 Verifying Trigonometric Identities

* Verify the identity:

$$\left(\tan\theta - \sec\theta\right)^2 = \frac{1-\sin\theta}{1+\sin\theta}$$

Ans: $$\left(\tan\theta - \sec\theta\right)^2 = \left(\frac{\sin\theta}{\cos\theta} - \frac{1}{\cos\theta}\right)^2 =$$

$$= \frac{(\sin\theta - 1)^2}{\cos^2\theta} = \frac{(1-\sin\theta)^2}{1-\sin^2\theta} = \frac{(1-\sin\theta)^{\cancel{2}}}{\cancel{(1-\sin\theta)}(1+\cos\theta)}$$

$$= \frac{1-\sin\theta}{1+\cos\theta}$$

* Verify the identity:

$$\frac{\sec^2(2u) - 1}{\sec^2(2u)} = \sin^2(2u)$$

Ans: $$\frac{\sec^2(2u) - 1}{\sec^2(2u)} = \frac{\sec^2(2u)}{\sec^2(2u)} - \frac{1}{\sec^2(2u)}$$

$$= 1 - \frac{1}{\frac{1}{\cos^2(2u)}} = 1 - \cos^2(2u) = \sin^2(2u)$$

* Verify the identity:

$$\sin^4 r - \cos^4 r = \sin^2 r - \cos^2 r$$

Ans: $\sin^4 r - \cos^4 r = \underbrace{(\sin^2 r + \cos^2 r)}_{=1}(\sin^2 r - \cos^2 r)$

$= 1 \cdot (\sin^2 r - \cos^2 r) = \sin^2 r - \cos^2 r$

* Verify the identity:

$$\frac{\cos\beta}{1-\sin\beta} = \sec\beta + \tan\beta$$

Ans: $\dfrac{\cos\beta}{1-\sin\beta} = \dfrac{\cos\beta}{1-\sin\beta} \cdot \dfrac{1+\sin\beta}{1+\sin\beta} =$

$= \dfrac{\cos\beta + \cos\beta\sin\beta}{(1-\sin\beta)(1+\sin\beta)} = \dfrac{\cos\beta + \cos\beta\sin\beta}{1-\sin^2\beta}$

$= \dfrac{\cos\beta + \cos\beta\sin\beta}{\cos^2\beta} = \dfrac{1}{\cos\beta} + \dfrac{\sin\beta}{\cos\beta}$

$= \sec\beta + \tan\beta$

* Verify the identity:

$$\log\cot x = -\log\tan x$$

Ans: $\log\cot x = \log\dfrac{\cos x}{\sin x} =$

$\log\cos x - \log\sin x =$

$$= -\left[\log \sin x - \log \cos x\right] =$$

$$= -\log \frac{\sin x}{\cos x} = -\log \tan x$$

* Verify the identity:

$$\frac{\cot\theta - \tan\theta}{\sin\theta + \cos\theta} = \csc\theta - \sec\theta$$

Ans: $$\frac{\cot\theta - \tan\theta}{\sin\theta + \cos\theta} = \left(\frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta}\right)\cdot\frac{1}{\sin\theta + \cos\theta}$$

$$= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}\cdot\frac{1}{\sin\theta+\cos\theta}$$

$$= \frac{(\cos\theta \cancel{+\sin\theta})(\cos\theta - \sin\theta)}{(\sin\theta\cos\theta)(\cancel{\sin\theta + \cos\theta})}$$

$$= \frac{\cos\theta}{\sin\theta\cos\theta} - \frac{\sin\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta} - \frac{1}{\cos\theta}$$

$$= \csc\theta - \sec\theta$$

* Show that $\sqrt{\sin^2 t} = \sin t$ is <u>not</u> an identity.

<u>I.e</u> find a value for which it is not satisfied. For instance at $t = \frac{3}{2}\pi$.