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1.2 Exponents and radicals

$n \in \mathbb{N}$ and $a \in \mathbb{R}$ then

<u>Exponents</u>

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

$\boxed{\text{Laws}}$:

* $a^m \cdot a^n = \underbrace{a \cdots a}_{m} \cdot \underbrace{a \cdots a}_{n} = a^{m+n}$
* $(a^m)^n = \underbrace{a^m \cdots a^m}_{n} = a^{mn}$
* $(ab)^n = \underbrace{(ab)(ab) \cdots (ab)}_{n} = a^n b^n$
* $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
* $\frac{a^m}{a^n} = a^{m-n} \qquad \frac{a^m}{a^n} = \frac{1}{a^{n-m}}$
* $\frac{a^{-m}}{b^{-n}} = \frac{b^n}{a^m} \qquad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$

<u>n-th root</u> $\quad n \in \mathbb{N} \quad n > 1 \quad a \in \mathbb{R}$

* $a = 0 \longrightarrow \sqrt[n]{a} = 0$
* $a > 0 \longrightarrow \sqrt[n]{a} = b > 0 \iff b^n = a$
* $\begin{cases} a < 0 \text{ and } n \text{ odd} & \sqrt[n]{a} = b < 0 \iff b^n = a \\ a < 0 \text{ and } n \text{ even} & \sqrt[n]{a} \text{ does not belong to } \mathbb{R} \end{cases}$

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$\sqrt[n]{a}$ is called radical , a = radicand

n = index of the radical

Laws : $\left(\sqrt[n]{a}\right)^n = a$

$\sqrt[n]{a^n} = a$ if $a \geq 0$

$\sqrt[n]{a^n} = a$ if $a < 0$ and n odd

$\sqrt[n]{a^n} = |a|$ if $a < 0$ and n even

* $\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$
* $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$
* $\sqrt[m]{\sqrt[n]{a}} = \sqrt[nm]{a}$

Finally :

$\frac{m}{n} \in \mathbb{Q}$ $n > 1$ $a \in \mathbb{R}$ then

* $a^{\frac{1}{n}} = \sqrt[n]{a}$
* $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$
* $a^{m/n} = \left(a^{\frac{1}{n}}\right)^m = \left(a^m\right)^{\frac{1}{n}}$

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Examples:

* $\dfrac{2^0 + 0^2}{2+0} = \dfrac{1+0}{2} = 1/2$

* $\left(-3/2\right)^4 - 2^{-4} = \dfrac{3^4}{2^4} - \dfrac{1}{2^4} = \dfrac{3^4-1}{2^4} = \dfrac{81-1}{16}$

  $= \dfrac{80}{16} = 5$

* $\dfrac{(2x^2)^3}{4x^4} = \dfrac{2^3 \cdot (x^2)^3}{4x^4} = \dfrac{8 \cdot x^6}{4 x^4} = 2x^2$

* $\left(\dfrac{4a^2b}{a^3b^2}\right)\left(\dfrac{5a^2b}{2b^4}\right) = \left(\dfrac{4}{ab}\right)\left(\dfrac{5a^2}{2b^3}\right) = \dfrac{10a}{b^4}$

  or $= \dfrac{20a^4b^2}{2a^3b^5} = 10\dfrac{a}{b^4}$

* $\left(\dfrac{c^{-4}}{16d^8}\right)^{3/4} = \left(\sqrt[4]{\dfrac{1}{16c^4d^8}}\right)^3 = \left(\dfrac{1}{2cd^2}\right)^3 =$

  $= \dfrac{1}{8c^3d^6}$ it's longer to do it $\sqrt[4]{(\quad)^3}$

* $\sqrt{x^2+y^2} = (x^2+y^2)^{1/2}$

* $(4x)^{3/2}$ = in radicals $= \left((2^2x)^3\right)^{1/2} =$

  $= \sqrt{2^6 \cdot x^3} = 8 \cdot \sqrt{x^3} = 8\sqrt{x^2 \cdot x} = 8x\sqrt{x}$

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* $\dfrac{1}{\sqrt[3]{2}}$ = rationalize = $\dfrac{1}{\sqrt[3]{2}} \cdot \dfrac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}$

$= \dfrac{\sqrt[3]{4}}{\sqrt[3]{2 \cdot 2^2}} = \dfrac{\sqrt[3]{4}}{2}$

* $\sqrt{\dfrac{1}{3x^3y}}$ = rationalize = $\dfrac{1}{\sqrt{3x^3y}} = \dfrac{1}{x\sqrt{3xy}} \cdot \dfrac{\sqrt{3xy}}{\sqrt{3xy}}$

$= \dfrac{\sqrt{3xy}}{x(3xy)} = \dfrac{\sqrt{3xy}}{3x^2y}$

* $\sqrt[4]{\dfrac{x^7y^{12}}{125x}} = \sqrt[4]{\dfrac{x^6y^{12}}{125}} = x y^3 \sqrt[4]{\dfrac{x^2}{5^3}}$

$= x y^3 \dfrac{\sqrt[4]{x^2}}{\sqrt[4]{5^3}} \cdot \dfrac{\sqrt[4]{5}}{\sqrt[4]{5}} = \dfrac{xy^3}{5}\sqrt[4]{5x^2}$

assume x, y might be negative

* $\sqrt{x^4y^{10}} = x^2\left|y^5\right|$

* $\sqrt[3]{a}\,\sqrt{a}$ = Combine into one radical

$= a^{1/3}\, a^{1/2} = a^{\frac{1}{3}+\frac{1}{2}} = a^{5/6} = \sqrt[6]{a^5}$

or $\sqrt[6]{a^2}\,\sqrt[6]{a^3} = \sqrt[6]{a^2a^3} = \sqrt[6]{a^5}$