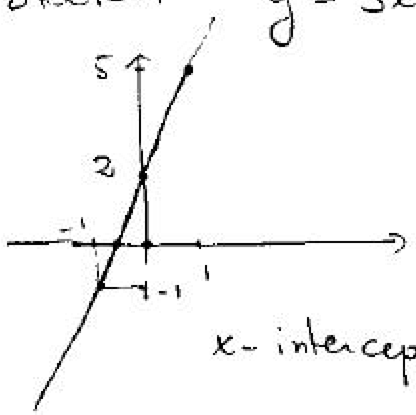
2.2 Graphs of equations

Graphs illustrate changes in quantities that are related by some equation. The graph gives a very intuitive and direct description.

* We start sketching the graph by plotting
* we'll want to pay attention to where the x- and y-intercepts are located
* If possible we'll pay attention to the symmetries as well (with respect to x or y axis and with respect to the origin).

Examples

* Sketch $y = 3x+2$



x	y
-3	-7
-1	-1
0	2
1	5

etc...

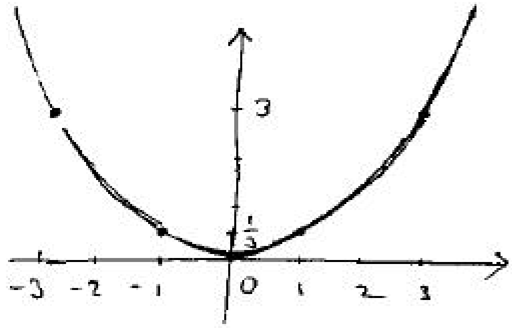
y-intercept is $(0,2)$

x-intercept is $y=0 \leadsto x = -2/3$

* Sketch $y = \frac{1}{3}x^2$.

x	y
± 0	0
± 1	$1/3$
± 3	3

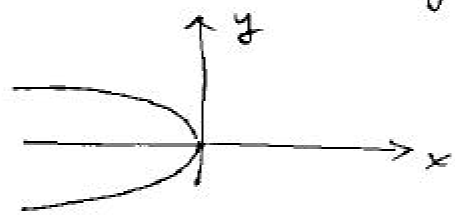
etc...



In this case we notice that the values $\pm x$ lead to the same y value

Such a function is said to be "even", the graph is symmetric w.r.t. the y-axis.

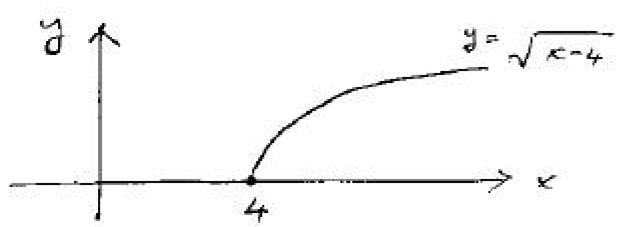
* $x = -2y^2$



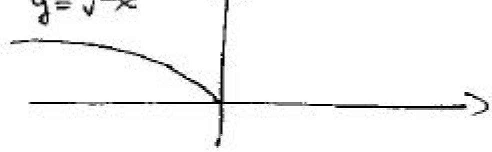
the graph is symmetric w.r.t. the x-axis

* $y = \sqrt{x-4}$ this is defined for $x - 4 \geq 0$ or $x \geq 4$. If we build a table of values

x	y
4	0
5	1
6	$\sqrt{2}$
8	2
$\vdots$	$\vdots$



* $y = \sqrt{-x}$

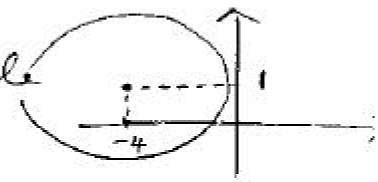


* write an equation for the set of points (x,y) that have distance 3 from $C(-4,1)$.

$$3 = d(P,C) = \sqrt{(x+4)^2 + (y-1)^2}$$

Square it: $(x+4)^2 + (y-1)^2 = 9$

This is the equation of a circle

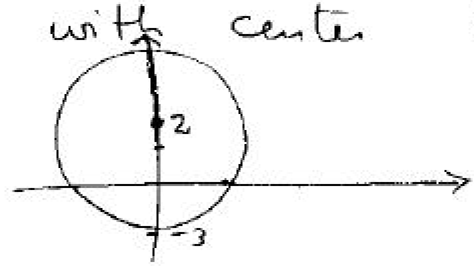


in general r = radius $C = (x_0, y_0)$ center

$$(x-x_0)^2 + (y-y_0)^2 = r^2$$

* Sketch $x^2 + (y-2)^2 = 25$.

Eq. of a circle with center $(0,2)$ and radius 5:



* Find the coordinates of the center and radius of the circle

$$9x^2 + 9y^2 + 12x - 6y + 4 = 0$$

$$(9x^2 + 12x \underbrace{+4}) + (9y^2 - 6y + 1) = -4 + 4 + 1$$

$$(3x+2)^2 + (3y-1)^2 = 1$$

$\therefore\ 9\left(x+\frac{2}{3}\right)^2 + 9\left(y-\frac{1}{3}\right)^2 = 1$

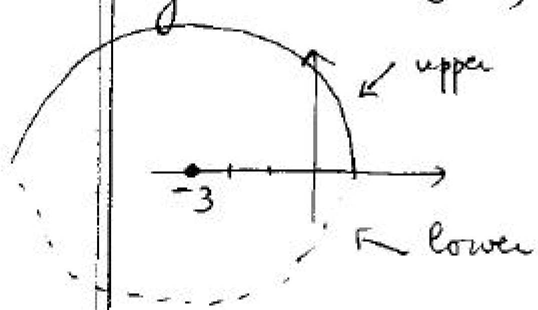
$\left(x+\frac{2}{3}\right)^2 + \left(y-\frac{1}{3}\right)^2 = 1/9$

$C\left(-2/3, \frac{1}{3}\right) \qquad r = 1/3$

* Find the equation of the upper (lower) half-circle if

$$(x+3)^2 + y^2 = 64$$

$y^2 = 64 - (x+3)^2 \rightsquigarrow y = \pm\sqrt{64-(x+3)^2}$



* Determine if $P(-2,5)$ is inside, outside or on the circle with center $C(3,7)$ and radius $r=6$.

Well, $d(P,C) = \sqrt{(3-(-2))^2 + (7-5)^2} =$

$= \sqrt{25+4} = \sqrt{29} < 6$

$\therefore$ P is inside.