Review for 1st Midterm

* Simplify: $\left(\dfrac{a^{2/3}b^{3/2}}{a^2 b}\right)^6 = \dfrac{\left(a^{2/3}b^{3/2}\right)^6}{(a^2 b)^6}$

$= \dfrac{(a^{2/3})^6 (b^{3/2})^6}{(a^2)^6 b^6} = \dfrac{a^4 b^9}{a^{12} b^6} = \dfrac{b^3}{a^8}$

* Simplify: $\sqrt{\sqrt[3]{(c^3 d^6)^4}} = \sqrt{\left(\sqrt[3]{c^3 d^6}\right)^4} =$

$= \sqrt{(c\, d^2)^4} = (c\, d^2)^2 = c^2 d^4$

* Simplify: $x^{-2} - y^{-1} = \dfrac{1}{x^2} - \dfrac{1}{y} = \dfrac{y - x^2}{x^2 y}$

* Simplify: $\dfrac{\dfrac{x}{x+2} - \dfrac{4}{x+2}}{x - 3 - \dfrac{6}{x+2}} = \dfrac{\dfrac{x-4}{x+2}}{\dfrac{(x-3)(x+2) - 6}{x+2}} =$

$= \dfrac{x-4}{\cancel{x+2}} \cdot \dfrac{\cancel{x+2}}{(x-3)(x+2) - 6} = \dfrac{x-4}{x^2 - x - 12} =$

$= \dfrac{\cancel{(x-4)}}{\cancel{(x-4)}(x+3)} = \dfrac{1}{x+3}$

* Factor: $u^3v^4 - u^6v =$

$= u^3v\,(v^3 - u^3) = u^3v\,(v-u)(v^2+uv+u^2)$

* Factor: $x^5 - 4x^3 + 8x^2 - 32 =$

$= x^3(x^2-4) + 8(x^2-4) =$

$= (x^2-4)(x^3+8) = (x-2)(x+2)(x+2)(x^2-2x+4)$

$= (x-2)(x+2)^2(x^2-2x+4)$

* Solve: $x(3x+4) = 5$

$3x^2 + 4x - 5 = 0$ Use quadratic formula

$$x_{1,2} = \frac{-4 \pm \sqrt{16 + 4\cdot 3\cdot 5}}{6} = \frac{-4 \pm \sqrt{76}}{6} =$$

$$= \begin{cases} \dfrac{-4 + 2\sqrt{19}}{6} = -\dfrac{2}{3} + \dfrac{1}{3}\sqrt{19} \\[2ex] \dfrac{-4 - 2\sqrt{19}}{6} = -2/3 - \dfrac{1}{3}\sqrt{19} \end{cases}$$

* Solve: $\dfrac{1}{x} + 6 = \dfrac{5}{\sqrt{x}}$

$x\left(\dfrac{1}{x} + 6\right) = x \cdot \dfrac{5}{\sqrt{x}}$

Recall
$x = \sqrt{x}\,\sqrt{x}$

$1 + 6x = 5\sqrt{x}$ or

$6x - 5\sqrt{x} + 1 = 0$

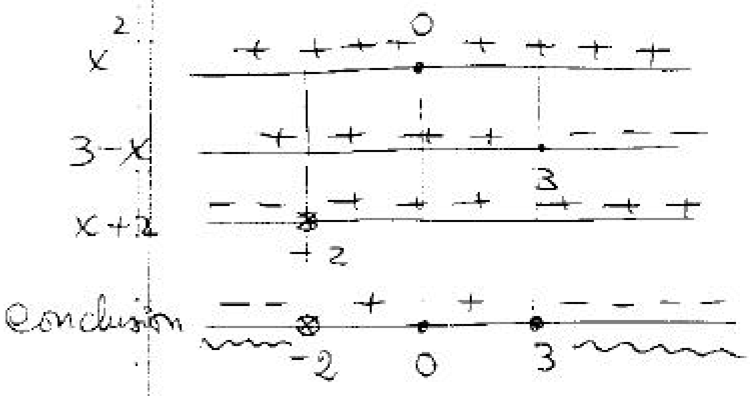
Set $y = \sqrt{x}$ or $y^2 = x$ So that

$6y^2 - 5y + 1 = 0$

$$\therefore \quad y_{1,2} = \frac{5 \pm \sqrt{25-24}}{12} = \frac{5 \pm 1}{12} = \begin{cases} \frac{6}{12} = \frac{1}{2} \\ \frac{4}{12} = \frac{1}{3} \end{cases}$$

Hence $x = y^2 \rightsquigarrow \begin{cases} x_1 = \frac{1}{4} \\ x_2 = \frac{1}{9} \end{cases}$

* Solve: $\dfrac{x^2(3-x)}{x+2} \le 0$



$(-\infty, -2) \cup \{0\} \cup [3, +\infty)$

↑ ↑ ↑

Careful !!

* Solve: $|16 - 3x| \ge 5$

This is equivalent to

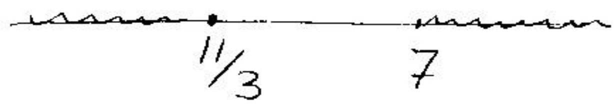
$16 - 3x \le -5$ and $16 - 3x \ge 5$

$-3x \le -5-16$ and $-3x \geq 5-16$

$-3x \le -21$ and $-3x \geq -11$

$x \geq 7$ and $x \le \frac{11}{3}$

$\therefore$



or $\left(-\infty, \frac{11}{3}\right] \cup [7, +\infty)$

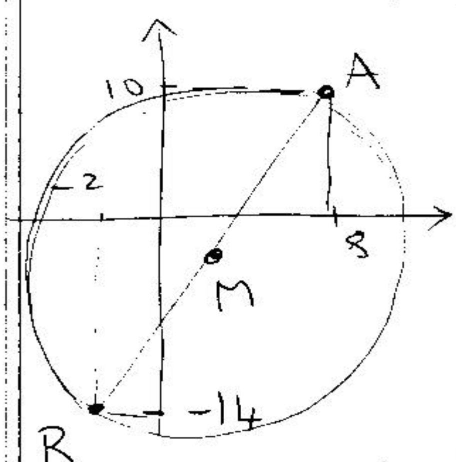
Express $(3+8i)^2$ in the form $a+ib$

$(3+8i)^2 = 3^2 + 2\cdot 3\cdot 8i + (8i)^2 =$

$= 9 + 48i + 64\cdot \underbrace{i^2}_{=-1} = 9-64+48i =$

$= -55+48i$

Find an equation of the circle that has endpoints of a diameter $A(8,10)$ and $B(-2,-14)$.



$M = \text{midpoint} = \text{Center circle}$

$= \left(\frac{8-2}{2}, \frac{10-14}{2}\right) = (3,-2)$

$\text{radius} = \frac{d(A,B)}{2} = d(A,M) = d(B,M)$

$d = (A,M) = \sqrt{(8-3)^2 + (10+2)^2} = \sqrt{25+144} = 13$

$\therefore$ equation of circle

$$(x-3)^2 + (y-(-2))^2 = 13^2$$

$$\boxed{(x-3)^2 + (y+2)^2 = 169}$$

* Find an equation of the line that has x-intercept -3 and passes through the center of the circle that has equation

$$x^2+y^2-4x+10y+26=0$$

The circle can be written as:

$$(x^2-4x \quad) + (y^2+10y \quad) = -26$$

We need to "complete the squares" in order to get the center of the circle

$$(x^2-4x+4) + (y^2+10y+25) = -26+4+25$$

$$\therefore (x-2)^2 + (y+5)^2 = 3$$

Center $C = (2,-5)$ $\quad$ (radius $= \sqrt{3}$) not relevant

Hence we want to find the Equation of

the line through $(-3, 0)$ and $(2, -5)$

$\uparrow$ x-intercept $\quad\quad$ $\uparrow$ center of circle

$$m = \text{slope} = \frac{0-(-5)}{-3-2} = \frac{5}{-5} = -1$$

Finally using $m=-1$ and $(-3, 0)$ we write the equation of the line

$$y - 0 = -1(x-(-3)) \qquad y = -x - 3$$

or $x + y = -3$

* Find $\dfrac{f(a+h)-f(a)}{h}$ if

$$f(x) = -x^2 + x + 5$$

$$f(a+h) = -(a+h)^2 + (a+h) + 5$$

$$f(a) = -a^2 + a + 5$$

Hence

$$f(a+h) - f(a) = \left[-(a+h)^2 + (a+h) + 5\right] - \left[-a^2 + a + 5\right]$$

$$= \cancel{-a^2} - 2ah - h^2 \cancel{+a} + h \cancel{+5} \cancel{+a^2} \cancel{-a} \cancel{-5}$$

$$= -2ah - h^2 + h$$

so that

$$\frac{f(a+h)-f(a)}{h} = \frac{-2ah-h^2+h}{h} = \boxed{-2a-h+1}$$

Find a linear function f such that

$f(1)=2$ and $f(3)=7$

Well, we are looking for the equation of the line through $(1,2)$ and $(3,7)$

slope $m = \dfrac{7-2}{3-1} = \dfrac{5}{2}$

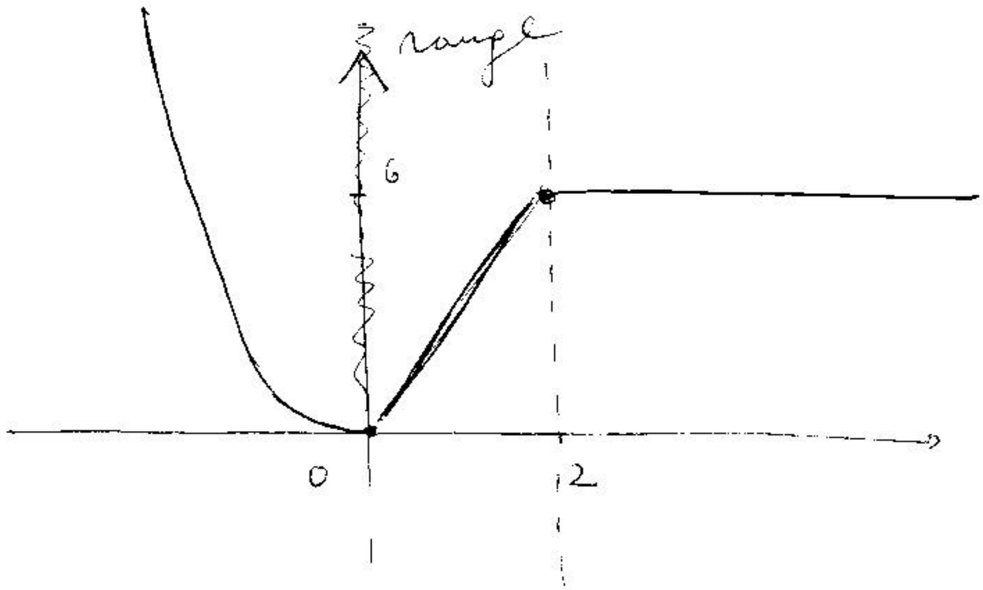
$y-2 = \frac{5}{2}(x-1)$ or $y = \frac{5}{2}x - \frac{5}{2} + 2$

so $y = \frac{5}{2}x - \frac{1}{2}$

i.e. $y = \boxed{f(x) = \frac{5}{2}x - \frac{1}{2}}$

Sketch the graph of

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 3x & \text{if } 0 \le x \le 2 \\ 6 & \text{if } x \geq 2 \end{cases}$$



The domain of f consists of all real number x.

The range of f consists of all $y \geq 0$ or $[0, +\infty)$

f is increasing on the interval $(0, 2)$

f is decreasing on the interval $(-\infty, 0)$

f is constant on the interval $(2, +\infty)$