3.1 Polynomial Functions of Degree Greater Than 2

We plan to study functions of the type

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Thus far we have considered the cases $n = 0, 1, 2$.

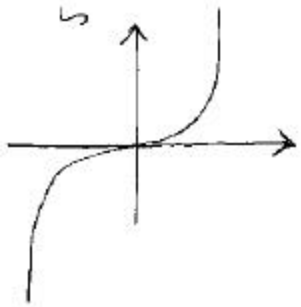
$n = 0$ $\quad y = a_0$ horizontal lines

$n = 1$ $\quad y = a_1 x + a_0$ any line

$n = 2$ $\quad y = a_2 x^2 + a_1 x + a_0$ any parabola

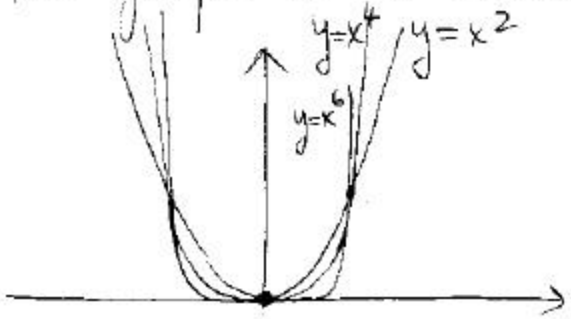
* When $n = 3$ we have a cubic. The simplest case is

$$y = x^3$$



* When $n = 4, 6, \ldots$ the graph is similar to a parabola:

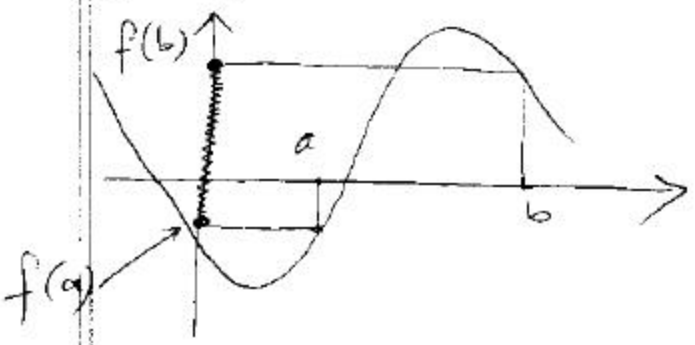
$$y = x^4, \quad y = x^6$$



They are more flat around $x = 0$ than $y = x^2$

Intermediate Value Theorem

If f is a polynomial function and $f(a) \neq f(b)$ for $a < b$ then f takes all values in between $f(a)$ and $f(b)$ in the interval $[a, b]$



* Ex: use the intermediate value theorem to show that $f(x) = 2x^3 + 5x^2 - 3$ has a zero between $x = -3$ and $x = -2$

We observe that $f(-3) = -12$ and $f(-2) = 1$. Since the graph of f has no breaks, it must cross the x-axis somewhere between -3 and -2. We have a zero for f at that intersection point!

* Sketch the graph of $y = 9x - x^3$ from its sign

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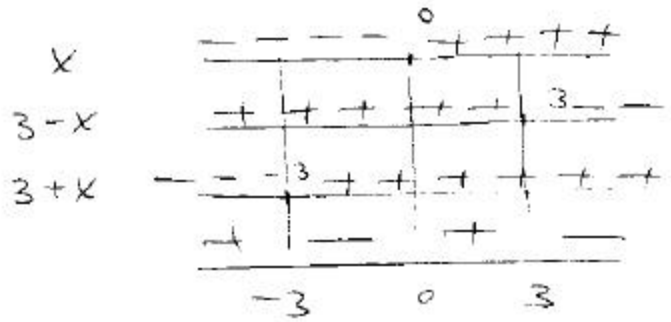
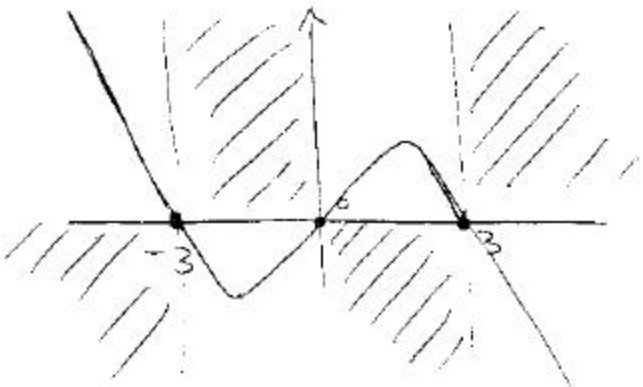
$$f(x) = 9x - x^3 = x(9 - x^2) = x(3-x)(3+x)$$



$y = 9x - x^3$



Ex: If $f(x) = kx^3 + x^2 - kx + 2$ find a number k such that the graph of f contains the point $(2, 12)$

$$12 = k \cdot 2^3 + 2^2 - k \cdot 2 + 2$$

$$12 = 8k + 4 - 2k + 2 \quad \rightsquigarrow \quad 6 = 6k$$

$$\therefore \quad \boxed{k = 1}$$

Ex: From a rectangular piece of cardboard having dimensions 20 inches by 30 inches, an open box is to be made by cutting out identical squares of area x^2 from each corner

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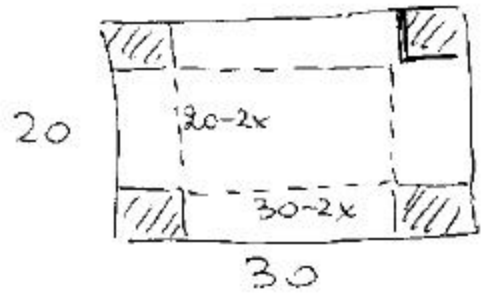
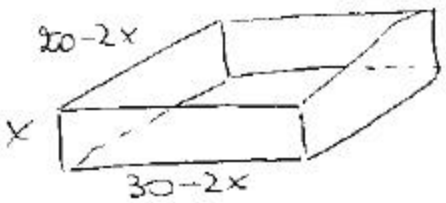
and turning up the sides.

(a) Find the volume of the box in terms of x

(b) Find all positive values of x such that $V(x) > 0$ and sketch the graph of V for $x > 0$.

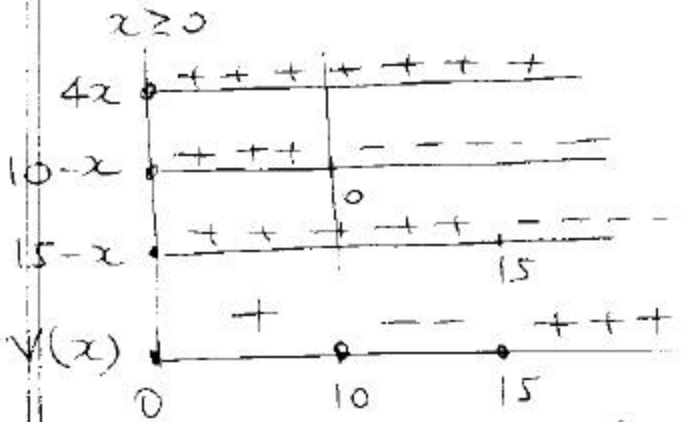
Ans:





Volume $= V(x) = x \cdot (20-2x)(30-2x)$

$= 4x(10-x)(15-x)$



Notice that x can be at most 10