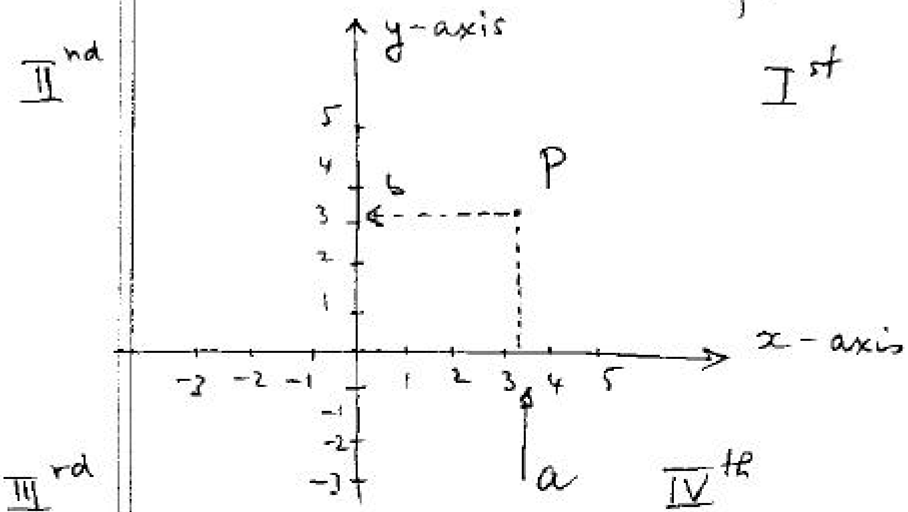
2.1 Rectangular Coordinate Systems

* There is a 1-1 correspondence between points in the plane and ordered pairs of real numbers. This occurs in the following manner:



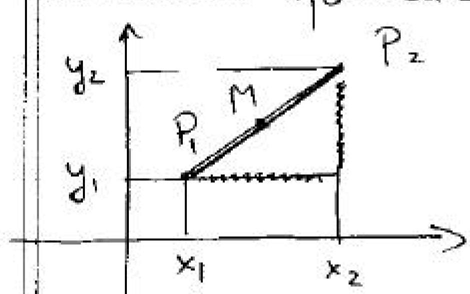
$P \longleftrightarrow (a, b)$

a = x-coordinate of P (abscissa)

b = y-coordinate of P (ordinate)

(a, b) are also called the Cartesian coordinates of P in honor of the philosopher Descartes.

* Distance formula between 2 points P_1 and P_2



Suppose $P_1(x_1, y_1)$ $P_2(x_2, y_2)$
THEN

$$\text{dist}(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

it follows from Pythagoras Thm.

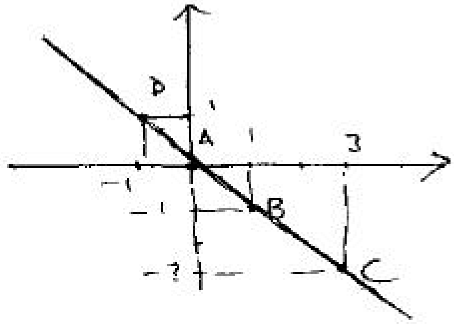
$$= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$= \text{dist}(P_2, P_1)$$

* Midpoint formula: $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$

Examples

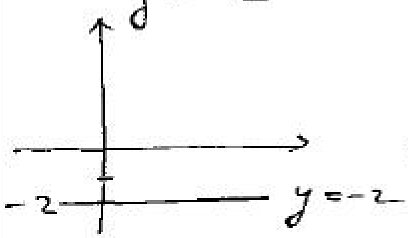
* Plot the points $A(0,0)$, $B(1,-1)$, $C(3,-3)$, $D(-1,1)$. Describe all points of the form $(a,-a)$, where $a \in \mathbb{R}$.



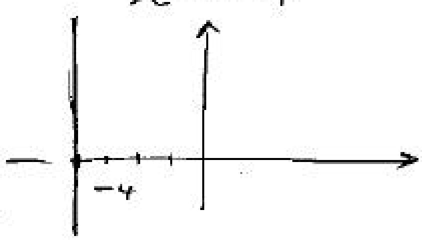
bisector of II and IV quadrant

* Describe all points $P(x,y)$ in a coordinate plane such that

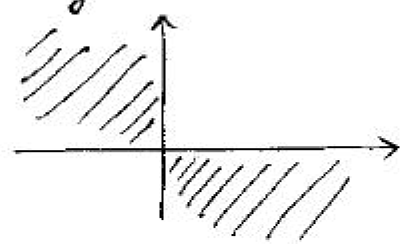
$y = -2$



$x = -4$



$\frac{x}{y} < 0$



* Find the distance $d(A,B)$ and the midpoint if $A(-4,7)$ and $B(0,-8)$

$$d(A,B) = \sqrt{(-4-0)^2 + (7-(-8))^2} = \sqrt{16 + 15^2} = \sqrt{241}$$

$$M = \left(\frac{-4+0}{2}, \frac{7-8}{2}\right) = \left(-2, -\tfrac{1}{2}\right)$$

* Show that the triangle with vertices $A(8,5)$, $B(1,-2)$, $C(-3,2)$ is a right triangle. Find its area.

$$d(A,B) = \sqrt{(1-8)^2 + (-2-5)^2} = \sqrt{49+49} = 7\sqrt{2}$$

$$d(A,C) = \sqrt{(-3-8)^2 + (2-5)^2} = \sqrt{121+9} = \sqrt{130}$$

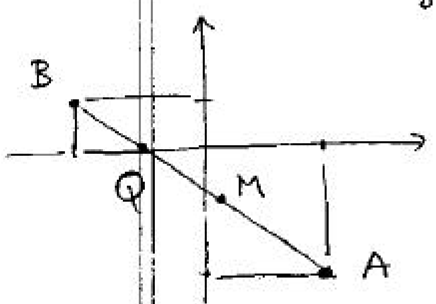
$$d(B,C) = \sqrt{(-3-1)^2 + (2-(-2))^2} = \sqrt{16+16} = 4\sqrt{2}$$

Notice that $\quad (4\sqrt{2})^2 + (7\sqrt{2})^2 = (\sqrt{130})^2$

$$32 + 98 = 130$$

Pythagoras Thm is verified!

* Given $A(5,-8)$ $B(-6,2)$ find the point on the segment AB that is three fourths of the way from A to B.



$$M = \left(\frac{5-6}{2}, \frac{-8+2}{2}\right) = \left(-\frac{1}{2}, -3\right)$$

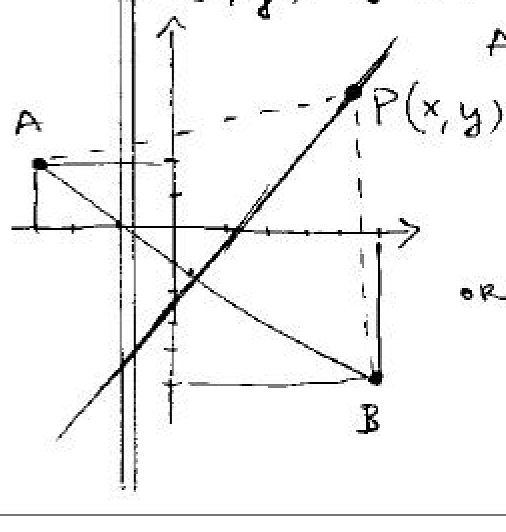
Q = midpoint of M and B

$$= \left(\frac{-1/2 - 6}{2}, \frac{2-3}{2}\right) = \left(-\frac{13}{4}, -\frac{1}{2}\right)$$

Other way: $Q = \left(\frac{3}{4}x_B + \frac{1}{4}x_A, \frac{3}{4}y_B + \frac{1}{4}x_A\right)$

* Find a formula that expresses the fact that $P(x,y)$ is on the perpendicular bisector ℓ of AB.

$A(-3,2)$ $B(5,-4)$



$$\text{dist}(A,P) = \text{dist}(B,P)$$

$$\sqrt{(x+3)^2 + (y-2)^2} = \sqrt{(x-5)^2 + (y+4)^2}$$

OR $\quad \cancel{x^2} + 6x + 9 + \cancel{y^2} - 4y + 4 = \cancel{x^2} - 10x + 25 + \cancel{y^2} + 8y + 16$

$$+16x - 12y = 41 - 13 = 28$$

$$\boxed{4x - 3y = 7}$$