MA 110 - 09/21/2005 FIRST MIDTERM	FALL 2005 Alberto Corso	Name: Answer Key

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

PROBLEM NUMBER	POSSIBLE POINTS	POINTS EARNED
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
TOTAL	out of 50	

1. Simplify the following expressions

(a) $\sqrt[3]{\sqrt[4]{(x^4y^8)^6}} =$ x^2y^4

$$\sqrt[3]{\left(\sqrt[4]{x^4y^8}\right)^6} = \sqrt[3]{(xy^2)^6} = (xy^2)^2 = x^2y^4$$

(b) $\frac{(x^6y^3)^{-1/3}}{(x^4y^2)^{-1/2}} =$ 1

$$\frac{(x^6)^{-1/3}(y^3)^{-1/3}}{(x^4)^{-1/2}(y^2)^{-1/2}} = \frac{x^{-2}y^{-1}}{x^{-2}y^{-1}} = 1$$

pts: /6

2. Factor completely the polynomial

$5x^3 + 10x^2 - 20x - 40 =$ $5(x+2)^2(x-2)$

$$= 5x^2(x+2) - 20(x+2)$$

$$= (x+2)\left[5x^2 - 20\right]$$

$$= 5(x+2)(x^2-4)$$

$$= 5\,\underbrace{(x+2)(x+2)}(x-2)$$

$$= \boxed{5(x+2)^2(x-2)}$$

pts: /6

3. Simplify the expression

$$\frac{5x}{2x+3} - \frac{6}{\underbrace{2x^2+3x}_{x(2x+3)}} + \frac{2}{x} = \frac{5x+4}{2x+3}$$

$$\frac{(5x)(x) - 6 + 2(2x+3)}{x(2x+3)}$$

$$= \frac{5x^2 - \cancel{6} + 4x + \cancel{6}}{x(2x+3)} = \frac{5x^2+4x}{x(2x+3)} = \frac{\cancel{x}(5x+4)}{\cancel{x}(2x+3)}$$

pts: /6

4. Solve the equation

$$x = 3 + \sqrt{5x-9}.$$

$$x - 3 = \sqrt{5x-9}$$

$$(x-3)^2 = \left(\sqrt{5x-9}\right)^2$$

$$(x-3)^2 = 5x - 9$$

$$x^2 - 6x + 9 = 5x - 9$$

$$x^2 - 11x + 18 = 0$$

$x =$ 9

$$(x-2)(x-9) = 0$$

$\cancel{x=2}$, $x = 9$

HOWEVER, $x=2$ is not a solution of the original equation.

$$2 \stackrel{?}{=} 3 + \sqrt{5\cdot 2 - 9}$$

$$2 \stackrel{?}{=} 3 + 1$$

$$2 \stackrel{?}{=} 4$$

No

pts: /6

5. Find the solutions of the equation

$$x^2 + 8x + 17 = 0$$

$$x_{1,2} = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 1 \cdot 17}}{2 \cdot 1} = \frac{-8 \pm \sqrt{64-68}}{2}$$

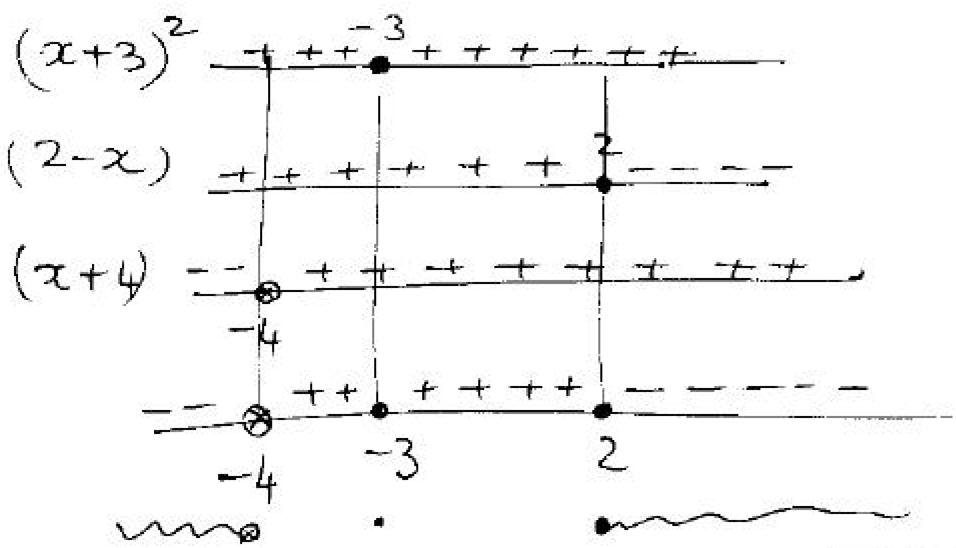
$$= \frac{-8 \pm \sqrt{-4}}{2} = \frac{-8 \pm 2i}{2} = -4 \pm i$$

$x =$ $-4+i$, $-4-i$

pts: /6

6. Solve the inequality

$$\frac{(x+3)^2(2-x)}{x+4} \le 0$$



answer : $(-\infty, -4) \cup \{-3\} \cup [2, +\infty)$

pts: /6

7. Solve the equation

$$|4x-1| = 7$$

$4x - 1 = 7$ or -7

$4x - 1 = 7 \longrightarrow x = 2$

$4x - 1 = -7 \longrightarrow x = -3/2$

$x =$ 2, $-3/2$

pts: /6

8. Simplify the difference quotient

$$\frac{f(2+h) - f(2)}{h},$$

if $f(x) = -2x^2 + 3$.

$f(2+h) = -2(2+h)^2 + 3 = -2(4 + 4h + h^2) + 3$

$= -8h - 2h^2 - 5$

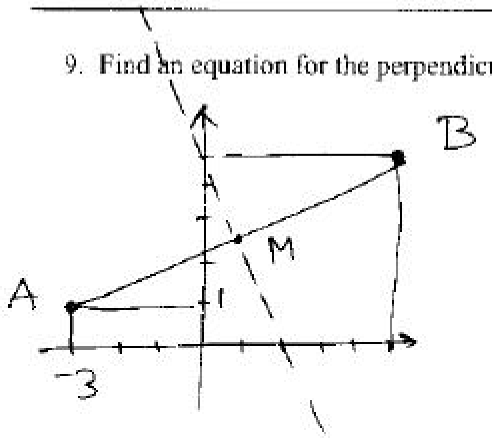
$f(2) = -2(2)^2 + 3 = -8 + 3 = -5$

$\therefore \dfrac{f(2+h) - f(2)}{h} = \dfrac{(-8h - 2h^2 - 5) - (-5)}{h}$

$= \dfrac{-8h - 2h^2}{h} = -8 - 2h$

pts: /6

9. Find an equation for the perpendicular bisector of the segment AB, where $A(-3, 1)$ and $B(5, 5)$.



$M = \text{midpoint} = \left(\frac{5-3}{2}, \frac{5+1}{2}\right)$
$= (1, 3)$

$\text{slope } AB = \frac{5-1}{5-(-3)} = \frac{4}{8} = \frac{1}{2}$

$\therefore$ slope bisector $= -2$

$\therefore$ equation of bisector $\quad y - 3 = -2(x-1)$

$\therefore \boxed{y = -2x + 5}$

pts: /6

10. Find the center and the radius of the circle with equation

$$x^2 + y^2 - 10x + 18 = 0.$$

$[x^2 - 10x \quad] + y^2 = -18$

add $+25$ $\quad +25$

$\therefore (x^2 - 10x + 25) + y^2 = 7$

$(x-5)^2 + y^2 = 7$

$\therefore$ C = center = $(5, 0)$ $\quad r$ = radius $= \sqrt{7}$

pts: /6