MA 110 - 10/19/2005 SECOND MIDTERM	FALL 2005 Alberto Corso	Name: Answer Key

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

PROBLEM NUMBER	**POSSIBLE POINTS**	**POINTS EARNED**
1.	6	
2.	6	
3.	6	
4.	6	
5.	6	
6.	6	
7.	6	
8.	6	
9.	6	
10.	6	
TOTAL	**out of 50**	

1. (*a*) Let $f(x) = \dfrac{x}{3x+2}$ and $g(x) = \dfrac{2}{x}$. Find $(f \circ g)(x)$.

$$f(g(x)) = \frac{2/x}{3\left(\frac{2}{x}\right)+2} = \frac{2/x}{\frac{6+2x}{x}} = \frac{2}{6+2x} = \frac{1}{3+x}$$

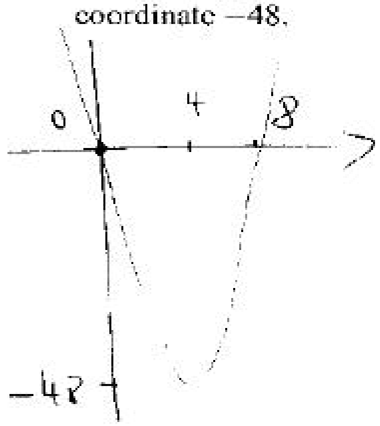
(*b*) Write $h(x) = \dfrac{\sqrt[3]{x}}{2-5\sqrt[3]{x}}$ as the composite of two functions $h_1(x)$ and $h_2(x)$.

$h = h_2 \circ h_1$, when $h_1(x) = \sqrt[3]{x}$

$$h_2(x) = \frac{x}{2-5x}$$

pts: /6

2. Find the equation of the parabola that has x-intercepts 8 and 0, and the lowest point has y-coordinate -48.



observe that the x-coordinate of the vertex is $\frac{0+8}{2} = 4$

$V(4, -48)$

hence

$$y = a(x-4)^2 - 48$$

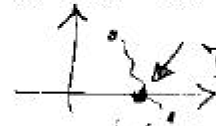
since $(0,0)$ in on the parabola:

$$0 = a(0-4)^2 - 48 \quad \therefore \quad a = \frac{48}{16} = 3$$

$$\boxed{y = 3(x-4)^2 - 48}$$

pts: /6

3. Use the intermediate value theorem to show that $f(x) = -x^4 + 3x^3 - 2x + 1$ has a zero in the interval $[2, 3]$.



root

$$f(2) = -2^4 + 3 \cdot 2^3 - 2 \cdot 2 + 1 = -16 + 24 - 4 + 1 = 5$$

$$f(3) = -3^4 + 3 \cdot 3^3 - 2 \cdot 3 + 1 = \cancel{-81} + \cancel{81} - 6 + 1 = -5$$

The graph of a polynomial functions has no breaks and goes (in this case) from a point with positive y-coordinate $[(2,5)]$ to a point with negative y-coordinate $[(3,-5)]$

Thus the graph must cross the y-axis

pts: /6

4. Use synthetic division to find the quotient and the remainder if $f(x) = 3x^5 - 38x^3 + 5x^2 - 1$ is divided by $p(x) = x - 4$.

	3	0	−38	5	0	−1
4		12	48	40	180	720
	3	12	10	45	180	719

i.e. $$f(x) = q(x) \cdot (x-4) + r(x)$$

with

$$q(x) = 3x^4 + 12x^3 + 10x^2 + 45x + 180$$

$$r(x) = 719$$

pts: /6

5. Use Descartes rule of signs to find the number of possible positive, negative, and non-real complex solutions of the equation $f(x) = 0$, where

$$f(x) = \underbrace{x^5 - 6x^4}_{\text{Yes}} \underbrace{+ 4x^2}_{\text{Yes}} + \underbrace{5x - 6}_{\text{Yes}}.$$

3 exchanges

$$f(-x) = -x^5 \underbrace{-6x^4 + 4x^2}_{\text{Yes}} \underbrace{- 5x}_{\text{Yes}} - 6$$

2 exchanges

positive zeros	3	1	3	1
negative zeros	2	2	0	0
non-real zeros	0	2	2	4
TOTAL	5	5	5	5

pts: /6

6. Find the zeros of $f(x) = (6x^2 + 7x - 5)^4(4x^2 - 1)^2$, and state the multiplicity of each zero.

$$= \left[(3x+5)(2x-1)\right]^4 \left[(2x-1)\ (2x+1)\right]^2$$

$$= (3x+5)^4 (2x-1)^4 (2x-1)^2 (2x+1)^2$$

$$= (3x+5)^4 (2x-1)^6 (2x+1)^2$$

$x = -5/3$	multiplicity 4
$x = 1/2$	multiplicity 6
$x = -1/2$	multiplicity 2

pts: /6

7. Show that the equation

$$x^3 + 5x - 3 = 0$$

has no rational root.

$f(x) = x^3 + 5x - 3$

a rational root is of the form $\frac{c}{d}$ where c divides -3 and d divides 1.

thus $\frac{c}{d} = \frac{\pm 1}{\pm 1}, \frac{\pm 3}{\pm 1} \rightsquigarrow \pm 1, \pm 3$

BUT $f(1) = 1 + 5 - 3 = 3 \neq 0$

$f(-1) = -1 - 5 - 3 = -9 \neq 0$

$f(3) = 27 + 15 - 3 = 39 \neq 0$

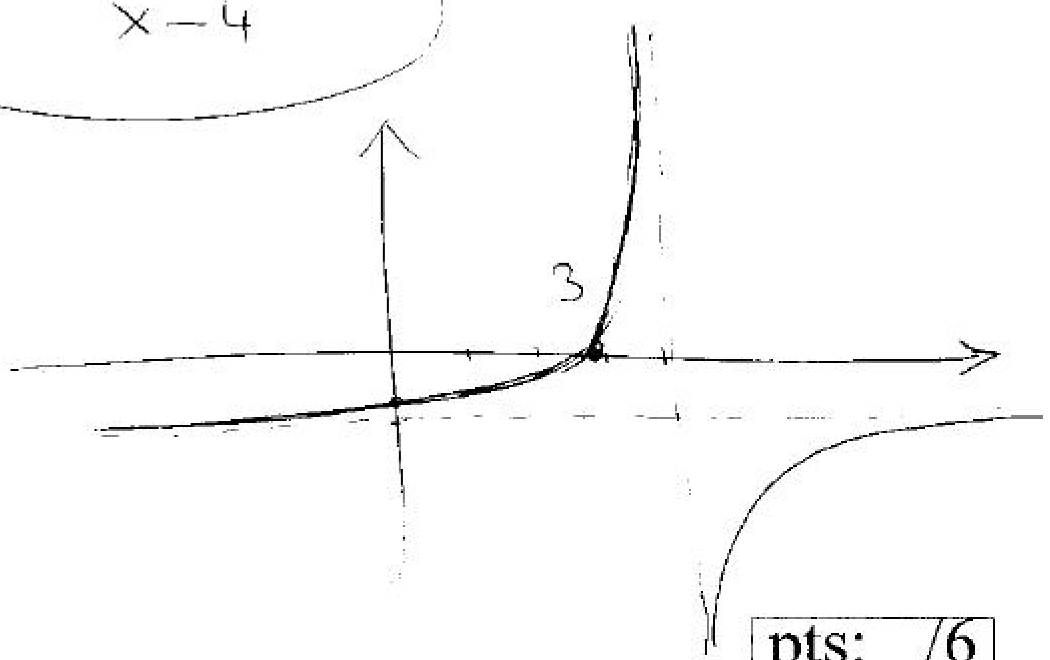
$f(-3) = -27 - 15 - 3 = -45 \neq 0$

pts: /6

8. Find an equation of a rational function $f(x)$ that has:

- vertical asymptote $x = 4$;
- horizontal asymptote $y = -1$;
- x-intercept 3.

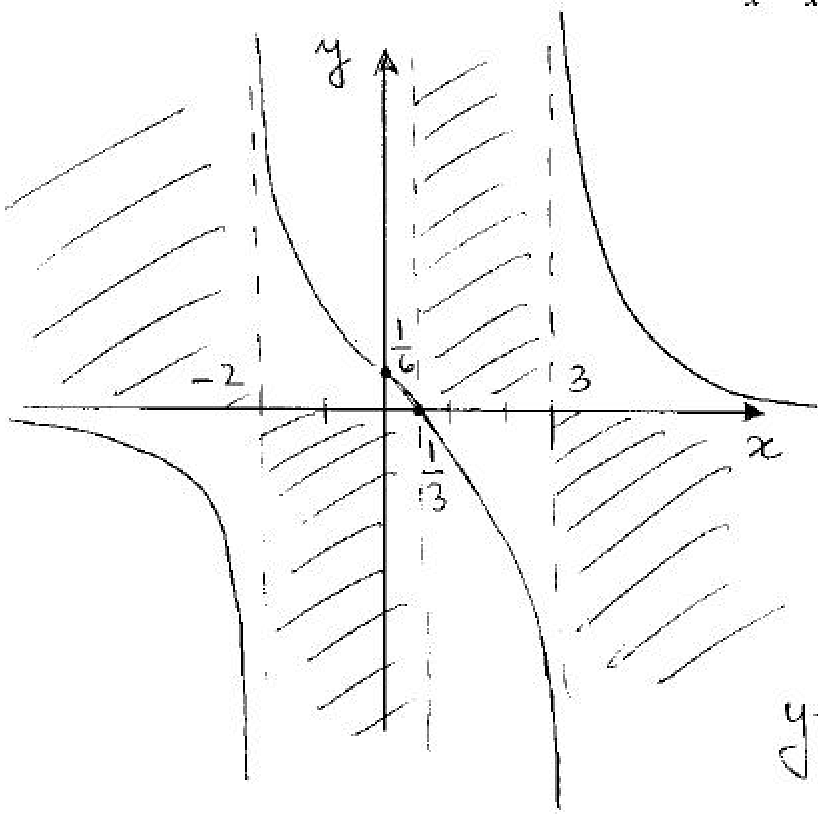
$$f(x) = \frac{3 - x}{x - 4}$$



pts: /6

9. Sketch the graph of

$$f(x) = \frac{3x-1}{x^2-x-6} = \frac{3x-1}{(x-3)(x+2)}$$



V.A.: $x = 3 \quad x = -2$

horiz A.: $y = 0$

Sign:

	$x<-2$	$-2<x<\frac{1}{3}$	$\frac{1}{3}<x<3$	$x>3$
$3x-1$	$- -$	$- -$	$+$	$+ + +$
x^2-x-6	$+ +$	$-$	$-$	$+ +$
	$-$	$+$	$-$	$+ +$

$-2 \qquad \frac{1}{3} \qquad 3$

y-intercept $\boxed{f(0) = \frac{1}{6}}$

pts: /6

10. A flat metal plate is positioned in an xy-plane such that the temperature T (in °C) at the point (x, y) is inversely proportional to the distance from the origin. If the temperature at the point $P(3, 4)$ is 20°C, find the temperature at the point $Q(24, 7)$.

$$T(x,y) = \frac{k}{\sqrt{x^2+y^2}}$$

$$T(3,4) = \frac{k}{\sqrt{3^2+4^2}} = 20 \rightsquigarrow \frac{k}{5} = 20$$

$$\therefore \quad k = 100$$

$$T(24,7) = \frac{100}{\sqrt{24^2+7^2}} = \frac{100}{\sqrt{625}} = \frac{100}{25} = 4$$

pts: /6