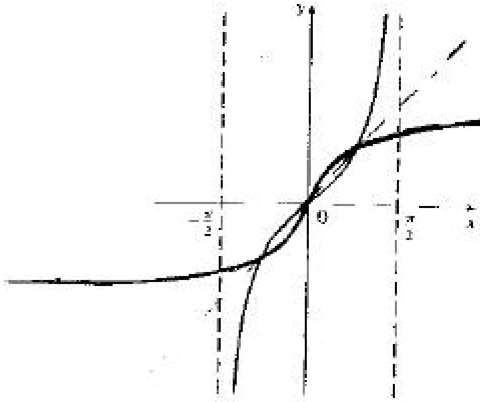
MA 110 - 11/16/2005 THIRD MIDTERM	FALL 2005 Alberto Corso	Name: Answer Key

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

PROBLEM NUMBER	POSSIBLE POINTS	POINTS EARNED
1	6	
2	6	
3	6	
4	6	
5	6	
6	6	
7	6	
8	6	
9	6	
10	6	
TOTAL	out of 50	

θ degrees	θ radians	$\cos\theta$	$\sin\theta$	$\tan\theta$
$0°$	0	1	0	0
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	0	1	—

1. (*a*) Determine if the graph given below is a graph of a one-to-one function f. If so, use the reflection property to sketch the graph of f^{-1}.



$f^{-1} = \tan^{-1} x$

(*b*) Find the inverse of $f(x) = \frac{2x+1}{3x}$.

$y = \frac{2x+1}{3x}$ $\quad 3xy = 2x+1 \quad 3yx - 2x = 1$

$x(3y-2) = 1 \quad x = \frac{1}{3y-2}$ invert $x \longleftrightarrow y$

$\therefore \boxed{y = \frac{1}{3x-2}}$

pts: /6

2. Find an exponential function of the form $f(x) = ba^x$ that has y-intercept at $\frac{1}{6}$ and passes through the point $P\left(2, \frac{32}{3}\right)$.

y-intercept at $\frac{1}{6}$ means $Q(0, \frac{1}{6})$ is on the graph. I.e. $\frac{1}{6} = b a^0 \Longrightarrow b = 1/6$

$\frac{32}{3} = \frac{1}{6} a^2 \iff a^2 = 64 \iff a = 8 \quad \boxed{f(x) = \frac{1}{6} \cdot 8^x}$

pts: /6

3. (*a*) Change to exponential form the following expressions:

$$\log \frac{1}{100} = -2 \qquad \log_a 343 = \frac{3}{4} \qquad \ln x = e.$$

$$10^{-2} = \frac{1}{100} \qquad a^{3/4} = 343 \qquad x = e^{e}$$

(*a*) Change to logarithmic form the following expressions:

$$3^5 = 243 \qquad 4^{5-x} = p \qquad (0.9)^t = \frac{1}{2}.$$

$$\log_3 243 = 5 \qquad \log_4 p = 5 - x \qquad \log_{0.9} \tfrac{1}{2} = t$$

pts: /6

4. Write the following expression as one logarithm:

$$\ln y^3 + \frac{1}{3}\ln(x^3y^6) - 5\ln y.$$

$$= \ln y^3 + \ln\left[(x^3y^6)^{1/3}\right] - \ln y^5$$

$$= \ln y^3 + \ln(xy^2) - \ln y^5$$

$$= \ln\left(\frac{y^3 \cdot xy^2}{y^5}\right) = \boxed{\ln(x)}$$

pts: /6

5. Solve the equation:

$$2^{5x+3} = 3^{2x+1}$$

$$\ln 2^{5x+3} = \ln 3^{2x+1}$$

$$(5x+3)\ln 2 = (2x+1)\ln 3$$

$$5x\ln 2 + 3\ln 2 = 2x\ln 3 + \ln 3$$

$$5x\ln 2 - 2x\ln 3 = \ln 3 - 3\ln 2$$

$$x\left[\ln 2^5 - \ln 3^2\right] = \ln 3 - \ln 2^3$$

$$x = \frac{\ln 3 - \ln 8}{\ln 32 - \ln 9} = \boxed{\frac{\ln(3/8)}{\ln(32/9)}}$$

pts: /6

6. Solve the equation:

$$\log_4(x) = \sqrt[3]{\log_4 x}.$$

$$u = \log_4 x \qquad \therefore \qquad u = \sqrt[3]{u} \iff u^3 = u$$

$$u^3 - u = 0 \iff u(u^2-1) = 0 \iff u(u-1)(u+1) = 0$$

$$\therefore \quad u = 0, \quad u = 1, \quad u = -1$$

$$\therefore \quad \log_4 x = 0, \quad \log_4 x = 1, \quad \log_4 x = -1$$

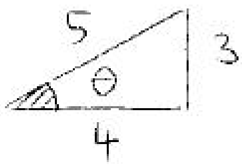
$$\Updownarrow \qquad\qquad \Updownarrow \qquad\qquad \Updownarrow$$

$$x = 1 \qquad\qquad x = 4 \qquad\qquad x = 4^{-1} = \frac{1}{4}$$

pts: /6

7. Find the exact values of the remaining trigonometric functions for the acute angle θ such that:

$$\tan\theta = \frac{3}{4}.$$



$\therefore$ $\cos\theta = 4/5$ $\sin\theta = 3/5$

$\cot\theta = 4/3$

$\sec\theta = 5/4$ $\csc\theta = 5/3$

pts: /6

8. Verify the identity

$$\sin\varphi(\csc\varphi - \sin\varphi) = \cos^2\varphi$$

by transforming the left-hand side into the right-hand side.

$$\sin\varphi\left(\csc\varphi - \sin\varphi\right) =$$

$$= \sin\varphi\left[\frac{1}{\sin\varphi} - \sin\varphi\right] =$$

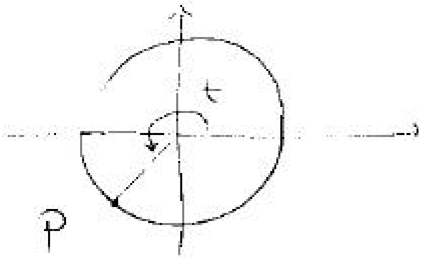
$$= 1 - \sin^2\varphi$$

$$= \cos^2\varphi \quad \checkmark$$

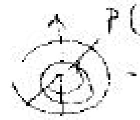
pts: /6

9. Let $P(t)$ denote the point on the unit circle that corresponds to the angle t.

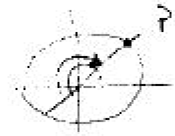
If $P(t)$ has coordinates $\left(-\frac{3}{5}, -\frac{4}{5}\right)$ find the coordinates of $P(t+3\pi)$, $P(t-\pi)$ and $P(-t)$.



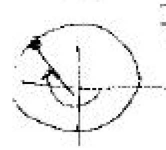
$P(t+3\pi) =$ $\left(\frac{3}{5}, 4/5\right)$



$P(t-\pi) =$ $(3/5, 4/5)$



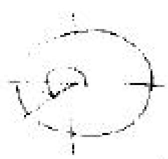
$P(-t) =$ $(-3/5, 4/5)$



pts: /6

10. Use reference angles to find the exact values of

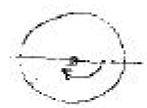
$\tan(225°)$ $\quad \sin\left(\frac{9}{2}\pi\right)$ $\quad \cos\left(-\frac{\pi}{6}\right)$ $\quad \sec(-150°)$



$= \tan(45°) = 1$

$\sin\left(\frac{\pi}{2} + 4\pi\right) = \sin(\pi/2) = 1$

$\cos(\pi/6) = \sqrt{3}/2$



$= \sec(180° + 30°) = \sec(210°) = \dfrac{1}{\cos(210°)} = -\dfrac{1}{\cos(30°)} = -\frac{2}{3}\sqrt{3}$

pts: /6