MA 110 - 12/13/2005 (Tuesday) FINAL EXAM (Practice Test)	**FALL 2005** Alberto Corso	Name: _____

Please, be neat and show all your work; circle your answer.

♣ Good Luck ♣

Problem Number	Possible Points	Points Earned	Problem Number	Possible Points	Points Earned
1.	6		**11.**	6	
2.	6		**12.**	6	
3.	6		**13.**	6	
4.	6		**14.**	6	
5.	6		**15.**	6	
6.	6		**16.**	6	
7.	6		**17.**	6	
8.	6		**18.**	6	
9.	6		**19.**	6	
10.	6		**20.**	6	
			TOTAL	**out of 100**	

Noteworthy values of trigonometric functions

Angle φ (↙ degrees, ↘ radians)		$\cos\varphi$	$\sin\varphi$	$\tan\varphi$
$0°$	0	1	0	0
$30°$	$\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
$60°$	$\frac{\pi}{3}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$
$90°$	$\frac{\pi}{2}$	0	1	—

The addition and subtraction formula

- $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
- $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$
- $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Double angle identities

- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$
- $\sin 2\alpha = 2\sin\alpha\cos\alpha$
- $\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

Half-angle identities

- $\cos\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1+\cos\alpha}{2}}$
- $\sin\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{2}}$
- $\tan\dfrac{\alpha}{2} = \pm\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$

where the choice of the sign is determined by the quadrant in which the angle $\alpha/2$ is located.

1. Let $A(3, -2)$ and $B(-3, 1)$. Write an equation for all points $P(x, y)$ such that

$$(\text{dist}(A, P))^2 = (\text{dist}(B, P))^2 + 1$$

pts: /6

2. Find b such that the line $3x + by + 2 = 0$ has y-intercept $1/2$.

pts: /6

3. Write an equation for the circle such that the points $A(6, 0)$ and $B(0, 8)$ are endpoints of a diameter.

pts: /6

4. Let $f(x) = x^2 - 6x + 8$. Sketch the graph of $y = f(x)$ using the method of completing the squares. Indicate the x- and y-intercepts as well as the coordinates of the vertex of the parabola.

pts: /6

5. Sketch the graph of

$$f(x) = \frac{x^2 - 5x + 4}{x^2}$$

Indicate domain, horizontal and vertical asymptotes, x- and y-intercepts, sign variation, etc...

pts: /6

6. Find the domain of

$$F(x) = \sqrt{\frac{1 - 2x}{3x + 2}}.$$

pts: /6

7. For $f(x) = -2x^2 + 7x - 1$, find the difference quotient

$$\frac{f(a+h) - f(a)}{h}.$$

pts: /6

8. Find the inverse of

$$f(x) = \sqrt{3 - x}.$$

Plot both the graph of $f(x)$ and the one of $f^{-1}(x)$ on the same coordinate plane. State the domain and the range of both functions.

pts: /6

9. Find the value of x for which:

$$\log_{3/2} x = 4; \qquad 4^x = 64; \qquad \log_x(\log_4 256) = 2.$$

pts: /6

10. The formula

$$p(h) = 14.7\, e^{-0.12h}$$

gives the atmospheric pressure in pounds per square inch at an altitude of h miles above sea level. At what altitude will the atmospheric pressure be one half of the sea level pressure?

pts: /6

11. Solve the logarithmic equation

$$\log_3(\log_2(x+3)) = \log_3(\log_2(4x-3)).$$

pts: /6

12. (a) Write in a single expression the following

$$4\log_3(x+1) - 2\log_3(x-1) - \frac{1}{2}\log_3(x+3).$$

(b) Write the following

$$\log\sqrt{\frac{x^2(x+1)^3}{(x+2)^5}}$$

in terms of $\log x$, $\log(x+1)$ and $\log(x+2)$.

pts: /6

13. If a central angle of 30° is subtended by a circular arc of length 12 meters, find the radius of the circle and the area of the sector.

pts: /6

14. If $\tan\theta = 4$ and $\sin\theta < 0$, find the remaining 5 trigonometric functions.

pts: /6

15. Find the amplitude, period and phase shift of

$$y = -2\sin(2(x - \pi/3)).$$

Graph the function and label the x- and y-intercepts.

pts: /6

16. (a) Verify the identity

$$\sec u = \frac{\cos u}{1 + \sin u} + \tan u$$

(b) Compute the exact value of

$$\frac{\cos(\pi/3)}{1 + \sin(\pi/3)} + \tan(\pi/3).$$

pts: /6

17. Solve the equation

$$\sin(2u) - \sqrt{3}\cos u = 0.$$

in the interval $[0, 2\pi]$.

pts: /6

18. Solve the triangle ABC if

$$\beta = 60^\circ \qquad \gamma = 45^0 \qquad a = 8.$$

pts: /6

19. Two joggers starting from the same point run along two directions that make an angle of 30°. One of them runs at 8mph and the other at 10mph. How far will they be after 2 hours?

pts: /6

20. If α and β are acute angles such that $\csc\alpha = 13/12$ and $\cot\beta = 4/3$, find

(a) $\sin(\alpha+\beta)$

(b) $\tan(\alpha+\beta)$

(c) the quadrant containing $\alpha+\beta$.

pts: /6