MA 113 - Calculus III | Fall 2003 | Name: ______________________ | Sec.: ___
THIRD MIDTERM | 11/18/2003

SEC.	INSTRUCTORS	T.A.'S	LECTURES	RECITATIONS
001	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 8:00-9:15, CB 205
002	A. Corso	D. Watson	MWF 8:00-8:50, CB 110	TR 9:30-10:45, CP 103
003	A. Corso	K. Messina	MWF 8:00-8:50, CB 110	TR 2:00-3:15, CP 287
004	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 8:00-9:15, BE 206
005	U. Nagel	E. Stokes	MWF 10:00-10:50, CP 220	TR 12:30-1:45, SRB 303
006	U. Nagel	K. Messina	MWF 10:00-10:50, CP 220	TR 3:30-4:45, CP 222

Answer all of the following questions. Use the backs of the question papers for scratch paper. No books or notes may be used. You may use a calculator. You may not use a calculator which has symbolic manipulation capabilities. When answering these questions, please be sure to:

- check answers when possible,
- clearly indicate your answer and the reasoning used to arrive at that answer (*unsupported answers may receive NO credit*).

QUESTION	SCORE	TOTAL
1.		10
2.		10
3.		15
4.		8
5.		10
6.		15
7.		10
8.		10
9.		12
Bonus.		5
TOTAL		100

1. Find all the critical values and the absolute maximum and absolute minimum values for

$$f(x) = 4x^3 - 15x^2 + 12x + 7$$

on the closed interval $0 \le x \le 3$.

$f'(x) = 12x^2 - 30x + 12 = 0$

$\iff 2x^2 - 5x + 2 = 0$

$\iff (2x-1)(x-2) = 0$

$x = \frac{1}{2}, \; x = 2$

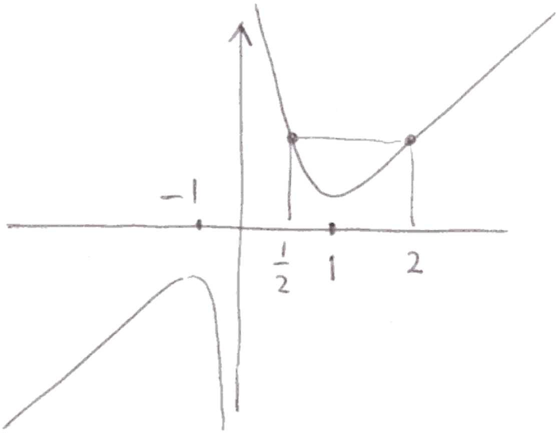
	x	$f(x)$
end points	0	7
	3	16 ←
critical values	$\frac{1}{2}$	9.75
	2	3 ←

abs max of 16 at $x = 3$

abs min of 3 at $x = 2$

pts: /10

2. (*a*) **Find** the value (or values) of c that satisfy the equation in the conclusion of the Mean Value Theorem for the function $f(x) = x + \frac{1}{x}$ defined on the interval $\frac{1}{2} \le x \le 2$.



$$\frac{f(2) - f(1/2)}{2 - 1/2} = \frac{5/2 - 5/2}{3/2} = 0$$

$f'(x) = 1 - \frac{1}{x^2} \quad \therefore \quad 1 - \frac{1}{x^2} = 0$

$\iff \frac{x^2 - 1}{x^2} = 0 \iff x^2 = 1 \quad x = \pm 1$

$x = 1$

(*b*) Suppose that the function $f(x)$ is differentiable on $[0,2]$ and that its derivative is never zero. **Show** that $f(0) \neq f(2)$.

By the MVT $\exists\, c \in (0,2)$ s/t

$f(2) - f(0) = f'(c)(2-0) = \underbrace{2f'(c) \neq 0}_{\text{by assumption}}$

$\therefore f(2) - f(0) \neq 0$

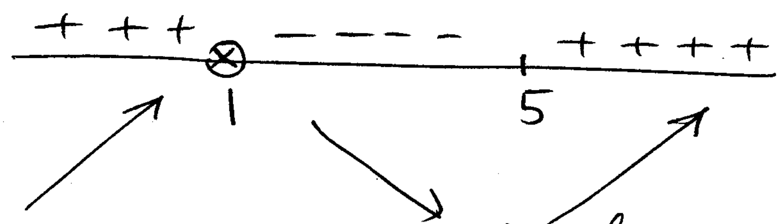
pts: /10

OR $f(2) \neq f(0)$

3. Consider the function: $$f(x) = \frac{2+x-x^2}{(x-1)^2}$$

It is also given that: $$f'(x) = \frac{x-5}{(x-1)^3} \quad \text{and} \quad f''(x) = \frac{2(7-x)}{(x-1)^4}$$

(*a*) (5pts) Determine the intervals where the graph of $f(x)$ is increasing or decreasing. Find the values of $f(x)$ at the local maxima and minima of $f(x)$.

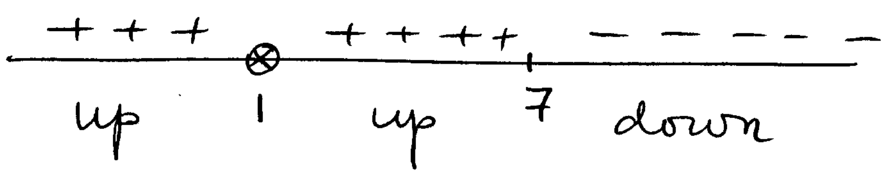


$f(5) = -\frac{18}{16} = -9/8$

local min at $x = 5$

(*b*) (5pts) Determine the intervals where the graph of $f(x)$ is concave up or down. Find the values of $f(x)$ at the inflection points of $f(x)$.

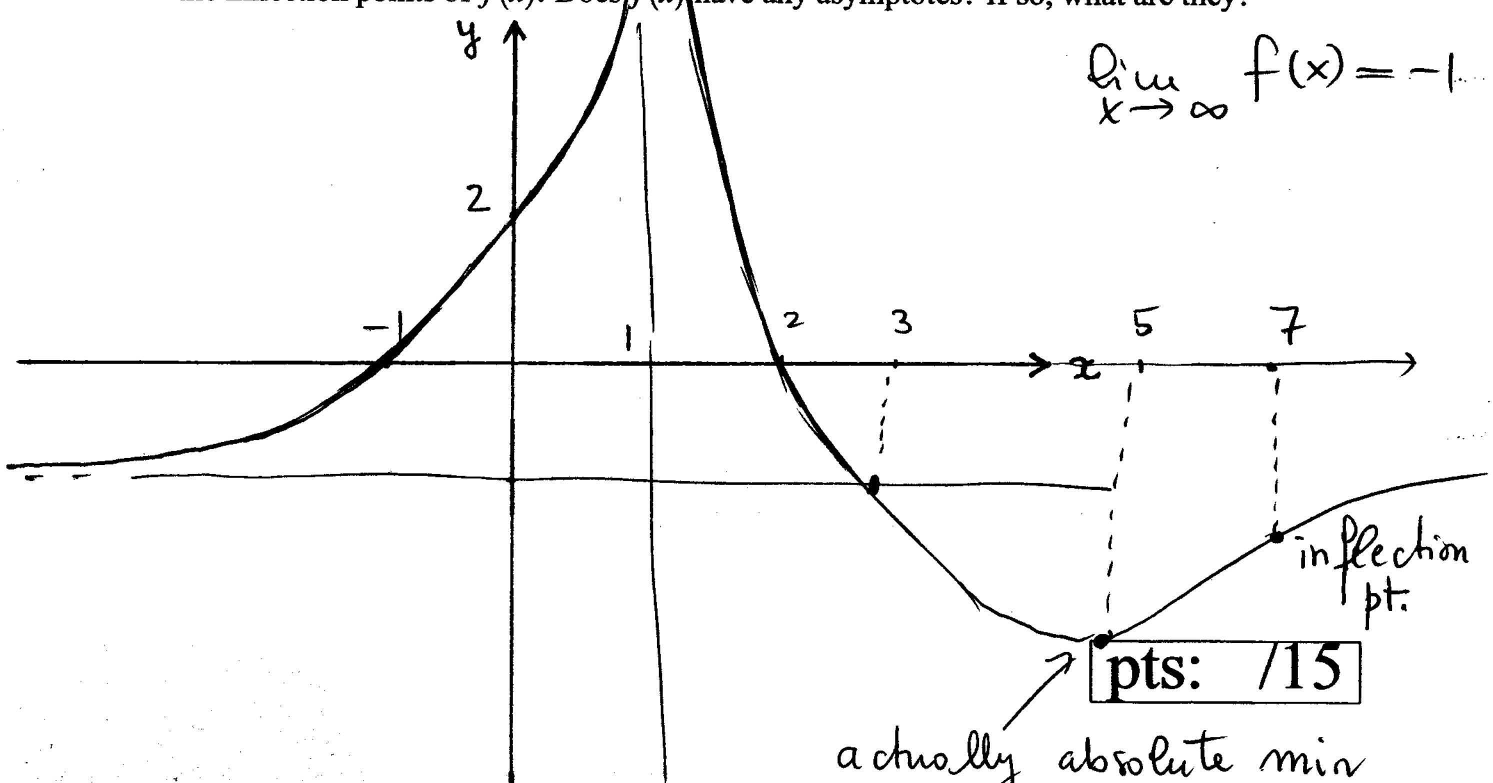
inflection point at $(7, f(7))$



$f(7) = \frac{-40}{36} = -\frac{10}{9}$

(*c*) (5pts) Sketch the graph of $f(x)$. **Make sure to** label the local maxima, the local minima and the inflection points of $f(x)$. Does $f(x)$ have any asymptotes? If so, what are they?

$\lim_{x \to \infty} f(x) = -1$



pts: /15

4. Without using a calculator, show that the equation

$$x^5 + 10x + 3 = 0$$

has exactly one real root.

existence

Let $f(x) = x^5 + 10x + 3$. If we apply the Intermediate value theorem on $[-1, 0]$ we have that the equation has a root between -1 and 0.

uniqueness

Suppose that it has 2 roots c_1, c_2. I.e. $f(c_1) = 0 = f(c_2)$. Since f is continuous and differentiable (everywhere) Rolle's Theorem apply. Hence $\exists\ c \in (c_1, c_2)$ s/t $f'(c) = 0$.

But $f'(x) = 5x^4 + 10 \neq 0$ always

CONTRADICTION

pts: /8

5. Show that if $x > 0$ then $x + \frac{4}{x} \geq 4$.

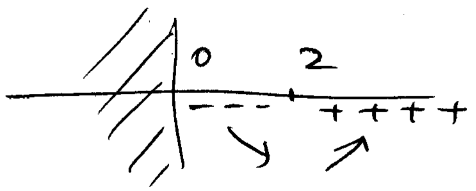
Let $f(x) = x + \frac{4}{x} = x + 4x^{-1}$

Consider $f'(x) = 1 - 4x^{-2} = 1 - \frac{4}{x^2} = \frac{x^2 - 4}{x^2}$

Its critical values on $(0, +\infty)$ are:

$$\frac{x^2 - 4}{x^2} = 0 \iff x^2 - 4 = 0 \iff x = \pm 2$$

$\boxed{x = 2}$



sign of f'

$\therefore$ $x = 2$ is actually an absolute min for $f(x)$

$\therefore$ $f(x) = x + \frac{4}{x} \geq f(2) = 2 + \frac{4}{2} = 4$

pts: /10

6. Each question is worth 5 points.

(a) $\lim_{x \to +\infty} \frac{2x+1}{x - x\sqrt{x}} = \underline{\quad 0 \quad}$

$$= \lim_{x \to +\infty} \frac{2 + \frac{1}{x}}{1 - \sqrt{x}} = \frac{2}{-\infty} = 0.$$

(b) $\lim_{x \to \infty} \frac{3\sqrt{1+16x^2}}{7-9x} = \underline{\quad -4/3 \quad}$

$$= \lim_{x \to \infty} \frac{3\sqrt{\frac{1}{x} + 16}}{7/x - 9} = \frac{3 \cdot 4}{-9} = -4/3$$

(c) Find the vertical and horizontal asymptotes of the curve

$$f(x) = \frac{2x^2+1}{2x-x^2}. = \frac{2x^2+1}{x(2-x)}$$

Compute $\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ for all the values of 'a' such that the line $x = a$ is a vertical asymptote of the given function $f(x)$.

$\lim_{x \to \pm\infty} f(x) = -2 \quad \therefore \quad y = -2$ is an horizontal asymptote

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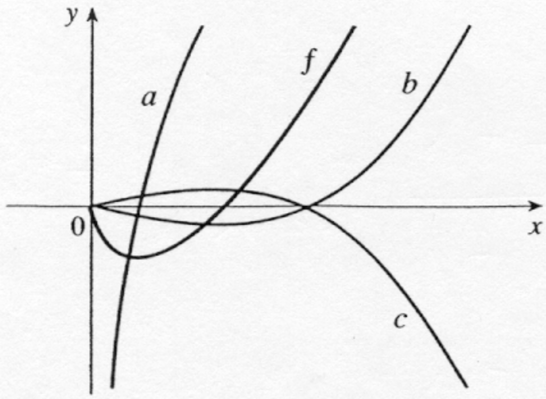
$f(x)$ has vertical asymptotes

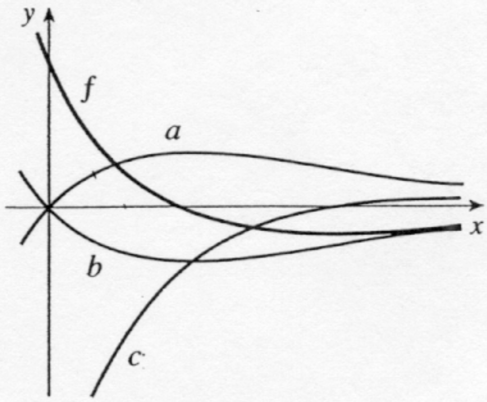
$x = 0, \quad x = 2$

$\lim_{x \to 0^+} f(x) = +\infty$	$\lim_{x \to 2^+} f(x) = -\infty$
$\lim_{x \to 0^-} f(x) = -\infty$	$\lim_{x \to 2^-} f(x) = +\infty$

pts: /15

7. (*a*) (4pts) The graph of a function $f(x)$ is shown. Which graph is an antiderivative of $f(x)$?





circle one:

a	*b*	*c*

circle one:

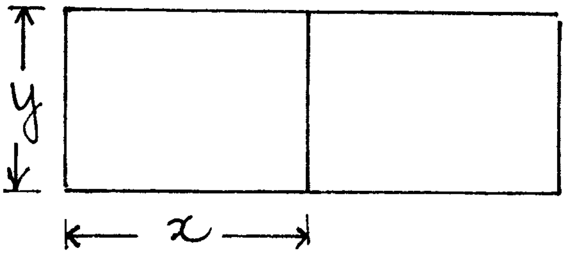
a	*b*	*c*

(*b*) (6pts) Find the most general antiderivative of: $f(x) = 2x^4 + \frac{1}{2\sqrt{x}} + 3\sin(6x)$.

$$F(x) = \frac{2}{5}x^5 + \sqrt{x} - \frac{1}{2}\cos(6x) + \text{Const}$$

pts: /10

8. A farmer wishes to fence two identical adjoining rectangular pens, each with 900 square feet of area, as shown in the picture



perimeter

$4x + 3y$

also : Area $= x \cdot y = 900$

$\therefore \; y = \frac{900}{x}$

What are x and y so that the least amount of fence is required?

We want to minimize the perimeter

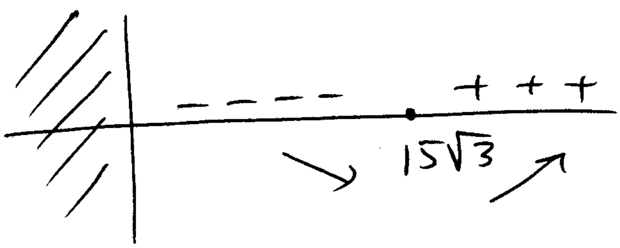
$$\boxed{\begin{array}{l} P(x) = 4x + 3 \cdot \frac{900}{x} \\ \text{subject to} \quad 0 < x < +\infty \end{array}}$$

$$P(x) = 4x + \frac{2700}{x}$$

$$P'(x) = 4 + 2700\left(-\frac{1}{x^2}\right) = \frac{4x^2 - 2700}{x^2}$$

$$P'(x) = 0 \iff 4x^2 - 2700 = 0 \iff x^2 = 675$$

$$\therefore \; x = +15\sqrt{3} \cong 25.98$$



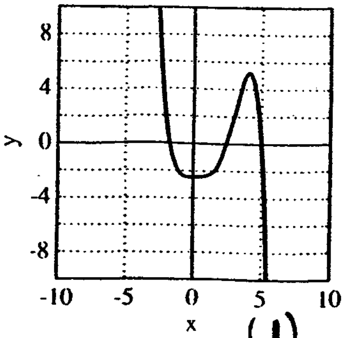
sign of $P'(x)$

$\therefore \; x = 15\sqrt{3}$ is the value we sought

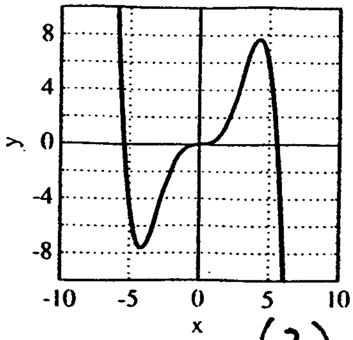
$y = 20\sqrt{3} \cong 34.64$

pts: /10

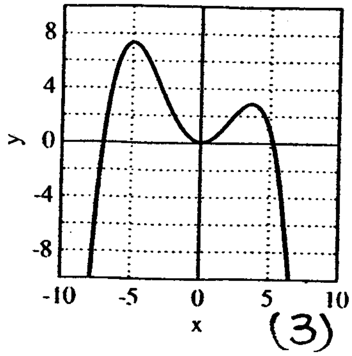
9. For each function $g(x)$ graphed in figures **(1)** – **(4)**



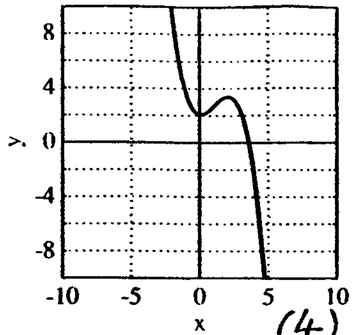
(1)



(2)

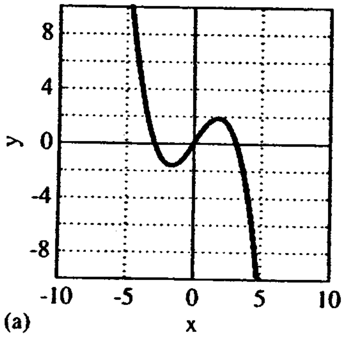


(3)

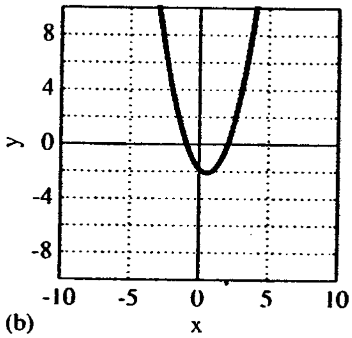


(4)

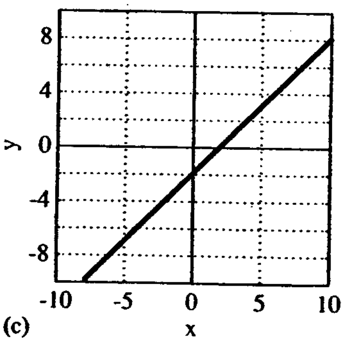
indicate which of the graphs in figures (a) – (f) is the graph of its **second** derivative.



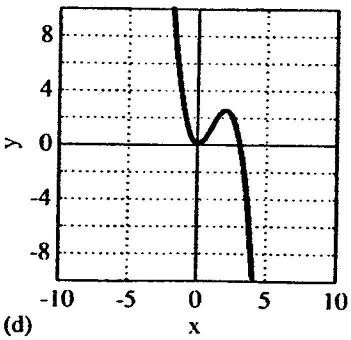
(a)



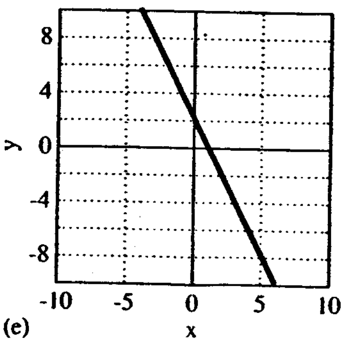
(b)



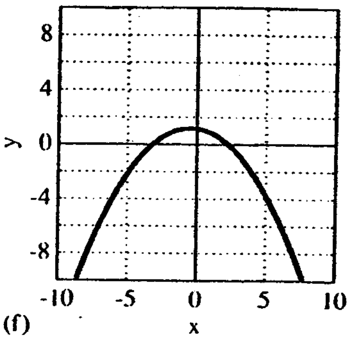
(c)



(d)



(e)



(f)

Answers:

$g(x)$	**(1)**	**(2)**	**(3)**	**(4)**
$g''(x)$	(d)	(a)	(f)	(e)

pts: /12

We have seen in class that

$$\sum_{i=1}^{n} 1 = n, \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}, \qquad \text{etc.}$$

Bonus. Evaluate the following limits:

(a) $\lim_{n\to+\infty} \frac{1+2+3+\cdots+n}{n^2} =$ $\underline{\quad 1/2 \quad}$

$$\lim_{n\to+\infty} \frac{\frac{n^2}{2}+\frac{n}{2}}{n^2} = 1/2$$

(b) $\lim_{n\to+\infty} \frac{1^2+2^2+3^2+\cdots+n^2}{n^3} =$ $\underline{\quad 1/3 \quad}$

$$\lim_{n\to\infty} \frac{\frac{2n^3}{6}+\cdots}{n^3} = 1/3$$

pts: /5