Math 115 Midterm # 1	Fall 95–Sections 27-30 Corso – Jones	Name: ________________ ________________	Please, be neat and show all your work; circle your answer.

1. Let $M(-8, 4)$ be the midpoint of the segment joining the points $A(3, n)$ and $B(m, -5)$.
   - Find the coordinates of A and B.
   - Write an equation for the line through M and perpendicular to the line through A and B.

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total: /50

2. Show that the lines

$$ax + by + c = 0 \quad \text{and} \quad bx - ay + d = 0$$

are perpendicular to each other.

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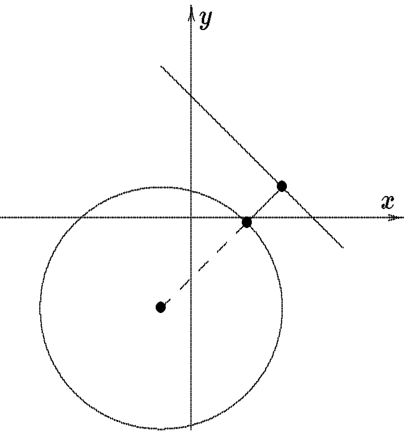
3. Find the center and the radius of the circle with equation

$$2x^2 + 2y^2 + 4x + 12y - 12 = 0.$$

Find the distance between the circle and the line $x + y - 4 = 0$ (see figure).

RECALL that, in general, the distance between the point $P(x_1, y_1)$ and the line ℓ with equation $ax + by + c = 0$ is given by

$$\mathrm{dist}(P, \ell) = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$



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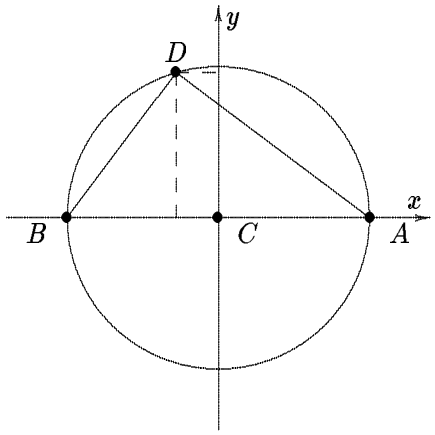
4. – Write an equation for the circle with center $C(0,0)$ and radius r.

– Write the coordinates of the points labelled in the pictures. (Hint: $D(x, ??)$).

– Using the tools of analytical geometry, show that the triangle ABD is a right triangle at D.



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5. Given

$$f(x) = \sqrt{x+2} \quad \text{and} \quad g(x) = \sqrt{3-x},$$

state the domain of $(f+g)(x)$ and $(f/g)(x)$.

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6. Use the various graphing techniques that you know to graph the following functions

- $f(x) = x - |x|$;
- $g(x) = \begin{cases} 1 & \text{if } x \leq 0 \\ 2 - x^2 & \text{if } x > 0. \end{cases}$

  What is $g(0)$?

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7. – Using the technique of "completing the squares", sketch the graph of the parabola

$$f(x) = 2x^2 + 12x + 10.$$

Make sure to list the vertex, the x- and y-intercepts and the axis of symmetry of the parabola.

– Let V, A, and B denote the vertex, and the two x-intercepts of the parabola $f(x) = 2x^2 + 12x + 10$. Find the area of the triangle VAB.

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8. The height $s(t)$ (in meters) above the ground after t seconds of a stone that has been thrown upward from ground level is given by

$$s(t) = -5t^2 + 35t.$$

- After how many seconds will the stone reach maximum height?
- Find the maximum height.

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9. - What does the difference quotient of a function represent geometrically?
   - Given the functions

$$f(x) = 2x^2 - 3x \quad \text{and} \quad g(x) = 2x^2 - 3x + 1$$

compute the difference quotients

$$\frac{f(a+h) - f(a)}{h} \quad \text{and} \quad \frac{g(a+h) - g(a)}{h},$$

where a and h are real numbers. Do you get two different answers? Explain your answer in geometric terms.

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10. Consider the quartic polynomial function

$$f(x) = x^2(2 - x)(x - 3).$$

- Factor completely the polynomial;
- find the x- and y-intercepts;
- determine the sign changes of the polynomial function as x varies along the x-axis;
- make a table of function values and plot the corresponding points;
- sketch the graph.

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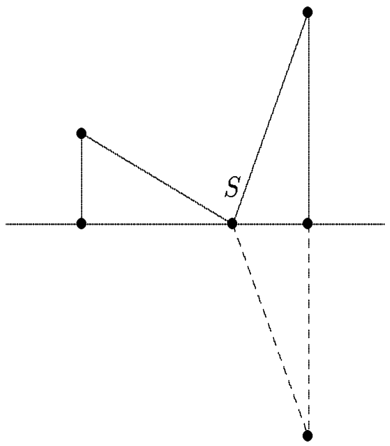
11. **(EXTRA CREDIT)** Two posts, one 12 feet high and the other 28 feet high, stand 30 feet apart. They are to be stayed by two wires, attached to a single stake S, running from ground level to the top of each post. Where should the stake be placed to use the least wire? (Hint: look the picture).



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