Precalculus

(Mathematics 640:115:Sec. 27-30 — Fall 1995)

FINAL: CAC - CAC GYM - Monday, December 18, 1995 - from 4:00pm to 7:00pm

1. Given $A(3,1)$ and $B(7,5)$, find the coordinates of the point M on the segment AB such that $dist(A,M) = 1/4\, dist(A,B)$.

2. Find the equation of the perpendicular bisector of AB, where $A(-1,3)$ and $B(3,-3)$.

3. Find all points $P(x,y)$ that satisfy the equation $x+y=0$ and that are $\sqrt{10}$ units from the point $(-3,5)$.

4. Find the centers and the radii of the circles

$$x^2+y^2=1 \qquad x^2+y^2-10x-2y+10=0.$$

   Plot them and find the distance between the two closest points on these two circles.

5. Let $f(x)=x^2-4x+1$.

   (a) Sketch the graph of $y=f(x)$ using the method of completing the squares (indicate x- and y-intercepts, and the vertex of the parabola).

   (b) Solve the inequality $x^2-4x+1 \geq 0$.

6. Sketch the graph of

$$f(x)=\frac{x^2}{x^2-5x+4}$$

   (indicate domain, horizontal and vertical asymptotes, x- and y-intercepts, sign variation, etc...).

7. Find the domain of

$$F(x)=\sqrt{\frac{x}{x-1}-1} \qquad f(x)=\frac{\sqrt{x}}{x^2-4}.$$

8. For $f(x)=3x^2-2x+5$, find the difference quotient

$$\frac{f(x+h)-f(x)}{h}.$$

9. Find the inverse of

$$f(x)=\frac{1-2x}{3x+2}.$$

   Plot both the graph of $f(x)$ and the one of $f^{-1}(x)$ on the same coordinate plane. State the domain and range of both functions.

10. Sketch the graph of the function

$$H(x) = \begin{cases} -x - 1 & if\ x < 0 \\ x^2 - 4 & if\ 0 \le x \le 2 \\ 2 & if\ x > 2. \end{cases}$$

11. Show that the product or quotient of two odd functions is an even function. Give an example.

12. (Extra Credit) Prove that every function can be written as the sum of an even function and an odd function.

13. Solve the followin equation

$$\log_3(2 \cdot 2^x + 4^x) = \log_3 24.$$

14. A vertical beam of light with intensity I_0 when hitting the ocean surface has intensity

$$I(d) = I_0 e^{-1.4d}$$

at a depth of d meters. At a depth of 2.5 meters below the ocean surface the intensity of the vertical beam is 8 lumen. What is the intensity of the beam at the ocean surface?

15. Sketch the graph of

$$y = 1 - (1/2)^{x-1}.$$

16. Find the x for which

$$3^x = 1/81; \qquad \log_x 27 = -3; \qquad \log_8 x = 5.$$

17. Suppose that $\log 3 = a$ and $\log 5 = b$. Express

$$\log_{15}\left(\frac{125}{9}\right)^{1/5}$$

in terms of a and b.

18. Find x if BD has length 1, $\alpha = 45°$, and $\beta = 60°$.

19. Find the amplitude, period, and phase shift of the function

$$y = 4\sin\left(x - \frac{\pi}{3}\right)\cos\left(x - \frac{\pi}{3}\right).$$

Graph the function.

20. Let α be an angle in the first quadrant such that $\sin\alpha = 5/13$, and β be an angle in the third quadrant such that $\cos\beta = -3/5$. Find $\cos(\alpha \pm \beta)$ and $\sin(\alpha \pm \beta)$.

21. Verify the identities

    (a) $\tan 2\theta = \dfrac{2\tan\theta}{2 - \sec^2\theta}$;

    (b) $\cos^4 x - \sin^4 x = 2\cos^2 x - 1$.

22. Find the exact values of

$$\cot(105^\circ), \quad \cos(\pi/8), \quad \sin(465^\circ), \quad \tan(-5/12\pi),$$

$$\sin(22^\circ 29' 60''), \quad \sin(\sin^{-1}(8/10) + \tan^{-1}(8/15)).$$

23. Solve the following trigonometric equations

    (a) $\sin 2x - \sin x = 0$ in $[0, \pi]$;

    (b) $\cos 2x = 1 - \sin x$ in $[-\pi/2, \pi/2]$.

24. Two observers located 250m apart at A and B on the bank of a river look at a point C on the opposite bank. If the angle CAB measures $46^\circ 30'$ and the angle ABC measures $53^\circ 20'$, find the distances from the two observers to the point C.

25. A regular pentagon is inscribed in a circle of radius 10m. How long is each side of the pentagon?

26. The hour hand of a clock is 5in long and the minute hand is 8in long. Find the distance between the outer ends of the hour and minute hands at 10 o'clock.