Ma123 Exam II
March 12, 2003

Instructions:
All submitted work is considered part of your answer. Be sure to cross out, erase, or otherwise indicate work that you do not want graded.

Show all work and explain your answers.
Unsupported answers will receive no credit.

There are 9 problems on 8 pages (including this cover page). Each problem counts 10 points. Check that you have a complete exam.

NO QUESTIONS WILL BE ANSWERED DURING THE EXAM

Fill in the information below and put your name or initials on each of the other pages.

Name: __________________________________________

Instructor: ______________________________________

Section Number/Class Meeting Time: _______________

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| Homework Percentage | 9 10 11 12 14 15 |
1. For each of the following calculate $f'(x)$ [DO NOT SIMPLIFY YOUR ANSWERS!]

a. $f(x) = (1-7x^2)^5$
   
   $f'(x) = 5(1-7x^2)^4 \cdot (-14x)$

b. $f(x) = \sqrt{x + \frac{1}{x}} = (x + \frac{1}{x})^{\frac{1}{2}}$
   
   $f'(x) = \frac{1}{2} (x + \frac{1}{x})^{\frac{1}{2} - 1} \cdot (1 - \frac{1}{x^2}) = \frac{1}{2} \cdot \frac{1}{\sqrt{x + \frac{1}{x}}} \cdot (1 - \frac{1}{x^2})$

Recall $\frac{d}{dx}[\frac{1}{x}] = -\frac{1}{x^2}$

2. Find each of the following for the function $f(x) = x^3 - 6x^2 + 9x + 1$

a. The critical number(s) for $f(x)$
   
   $f'(x) = 3x^2 - 12x + 9 = 0 \iff x^2 - 4x + 3 = 0 \quad (x-3)(x-1) = 0$
   
   $\therefore x = 3, x = 1$

b. The interval(s) on which $f(x)$ is decreasing
   
   \[
   \begin{array}{cccccc}
   \text{sign of } f'(x) & + & + & - & - & +
   \\
   x & 0 & 3 & 4
   \end{array}
   
   \therefore f(x) \text{ is decreasing on } (1, 3)

c. The interval(s) on which $f(x)$ is concave down
   
   $f''(x) = 6x - 12 = 0 \iff x = 2$
   
   \[
   \begin{array}{cccccc}
   \text{sign of } f'' & + & + & + & + & +
   \\
   x & 0 & 2 & 4
   \end{array}
   
   \therefore f(x) \text{ is concave down on } (-\infty, 2)$

d. The value(s) of $x$ at which $f(x)$ has a local minimum
   
   By the work in (b) there is a local minimum for $x = 3$

e. The point(s) of inflection of the graph of $f(x)$ [be sure to give both coordinates].
   
   There is an inflection point at
   
   \[(2, f(2)) = (2, 3)\]
3. Find the values of A and B in the following:

a. Find the value of A so that \( f(x) = Ax^3 - 12x^2 \) has a critical point at \( x = 1 \).

We want \( f'(1) = 0 \). Now, \( f'(x) = 3Ax^2 - 24x \)

So \( f'(1) = 3A - 24 = 0 \) \( \uparrow \) \text{want} \( A = 8 \)

b. Find the value of B so that \( f(x) = Bx^3 - 12x^2 + 7x \) has a point of inflection at \( x = -1 \).

\[ f'(x) = 3Bx^2 - 24x + 7 \quad f''(x) = 6Bx - 24 \]

\text{Want} \( f''(-1) = 0 \) \( \leftrightarrow -6B - 24 = 0 \) \( \therefore B = -4 \)

Notice that for \( B = -4 \) the sign of \( f''(x) \) is: \( +++ \rightarrow -1 \)

4. A computer store can buy packages of CD blanks for $1.00 per package. If they charge $3.00 per pack then they can sell 600 packages per month. For each $.10 that they reduce the selling price they can sell an additional 60 packages per month. At what price should they sell the packages in order to realize the maximum profit?

\( \text{don't do it yet} \)
5. The following is the graph of the derivative, \( f'(x) \), of a function \( f(x) \). Express the following in terms of A, B, C, D, and E.

This is the graph of the derivative of \( f(x) \)

a. All subinterval(s) of \([A, E]\) where \( f(x) \) is increasing.

\[
\begin{bmatrix}
A & E
\end{bmatrix}
\]

b. All subinterval(s) of \([A, E]\) where \( f(x) \) is concave up

\[
(A, C)
\]

as the sign of \( f'' \):

\[
\begin{array}{c}
A & C
\end{array}
\]

\[
E
\]

there is a local max at \( x = E \)

c. The point(s) at which \( f(x) \) has a local maximum.

\[
\text{sign of } f'(x)
\]

\[
E
\]

d. The x-coordinate(s) of all inflection point(s) of \( f(x) \).

\[
\text{sign of } f''
\]

\[
A & C
\]

\[
\text{inflection point at } x = A, \ x = C
\]

6. Find the absolute maximum of \( f(x) = x + \frac{4}{x} \) on the interval \([1, 6]\).

\( f \) is a continuous function over a closed interval so that the absolute max can occur either at the end points or a the critical values inside the interval.

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<td>1</td>
<td>5</td>
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So, \( \text{abs max} = 10.66 \) or \( \frac{20}{3} \)
7. Suppose \( f(x) \) is a differentiable function with \( f(8) = 3 \) and \( f'(8) = -1 \). If \( h(x) = f(x)^3 \) then:

a. Calculate \( h'(8) \)

\[
h'(x) = 3 f(x)^2 \cdot f'(x) \quad \Rightarrow \quad h'(8) = 3 \cdot (3)^2 \cdot (-1)\]

Use generalized power rule

\[
= -27
\]

b. Calculate the equation of the tangent line to the graph of \( h(x) \) at \( x = 8 \).

\[
P(8, h(8)) \quad \text{now} \quad h(8) = (f(8))^3 = 3^3 = 27
\]

\[
m = h'(8) = -27
\]

\[
(y - 27) = -27(x - 8)
\]

or \( y = -27x + 243 \)

8. A projectile travels along the x-axis in such a way that its distance from 0 at time \( t \) is given by the formula \( x(t) = 12t - 4t^2 \).

a. In which direction (left or right) is the projectile traveling at \( t = 2 \)?

\[
\nu(t) = x'(t) = 12 - 8t
\]

\[
\nu(2) = 12 - 8 \cdot 2 = -4\]

so it is travelling to the left

b. What is the velocity of the projectile at \( t = 1 \)?

\[
\nu(1) = 12 - 8 \cdot 1 = 4
\]

c. At which time(s) does the projectile stop?

\[
\nu(t) = 0 \quad \Leftrightarrow \quad 12 - 8t = 0 \quad \Leftrightarrow \quad t = \frac{12}{8} = \frac{3}{2} = 1.5
\]

d. At which time(s) is the projectile at \( x = 0 \)?

Need to solve \( x(t) = 12t - 4t^2 = 0 \) the equation factors

\[
4t(3 - t) = 0
\]

\[
\therefore \quad t = 0 \quad \text{and} \quad t = 3
\]
9. Sketch possible graphs of the functions described below.

A. Sketch the graph of a function $f(x)$ which satisfies:

a. $f(x) > 0$ on the interval $(-5, 1)$ and on the interval $(3, 5)$

b. $f'(x) < 0$ on the interval $(1, 3)$

---

Notice that I could have also graphed the function because in (a) they asked for $f(x) > 0$ not $f'(x) > 0$
B. Sketch the graph of a function \( f(x) \) which satisfies:

c. \( f''(x) > 0 \) on the interval \((-5, 4)\)

d. \( f''(x) < 0 \) on the interval \((4, 5)\)

I could have chosen to graph a discontinuous function at \( x = 4 \)
C. Sketch the graph of a function $f(x)$ which is defined on the interval (-5,5) and which satisfies:

- f(x) is not differentiable at $x = -2$
- $f'(3) = 0$

Another possibility: