MTH 133 - 9/29/98 FIRST MIDTERM	Fall 98 A. Corso - A. Minut	Name: _____

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	15	
2	15	
3	10	
4	15	
5	15	
6	15	
7	15	
TOTAL	100	

1. Find the derivative of the following functions:

(a) $y = \ln(\ln(\ln x))$

(b) $y = \sqrt[3]{\dfrac{x(x+1)(x-2)}{(x^2+1)(2x+3)}}$ (use logarithmic differentiation)

(c) $y = \ln\left(\dfrac{e^x}{1+e^x}\right)$

pts: /15

2. Find the following integrals:

(a) $\int_0^{\sqrt{\ln \pi}} 2xe^{x^2} \cos e^{x^2}\, dx$

(b) $\int_2^{16} \frac{dx}{2x\sqrt{\ln x}}$

(c) $\int \frac{2^{\ln x}}{x}\, dx$

pts: /15

3. Suppose $f(x) = \ln(4 - x^2)$.

   (a) For which values of x is $f(x)$ defined?

   (b) For which of the values of x found in part (a) is $f(x)$ increasing (i.e. the first derivative is positive)?

pts: /15

Newton's Law of Cooling states that the rate at which an object changes temperature is proportional to the difference between its temperature and the temperature of the surrounding environment. In other words, if T_S is the (constant) temperature of the environment, the temperature $T(t)$ of the object at time t satisfies

$$\frac{d}{dt}T(t) = -k(T(t) - T_S)$$

for some constant k. Let T_0 denote the temperature of the object at time $t = 0$. It was shown in class that the function $T(t)$ is given by

$$T(t) = T_S + (T_0 - T_S)e^{-kt}.$$

4. How long will it take an object at $100^\circ C$ in a room of constant temperature $20^\circ C$ to cool to $30^\circ C$ if it takes 10 minutes for it to cool to $50^\circ C$?

pts: /15

5. Let R be the region bounded by the parabola $y = x^2 + 1$ and the line $y = x + 3$.

   (a) FIND the area of the region R.

   (b) Set up but DO NOT EVALUATE a definite integral to find the volume of the solid generated by revolving R about the line $y = 5$.

   (c) Set up but DO NOT EVALUATE a definite integral to find the volume of the solid whose cross sections perpendicular to the x-axis are equilateral triangles.

pts: /15

6. Find the length of the curve

$$y = \int_0^x \sqrt{\cos(2t)}\, dt$$

from $x = 0$ to $x = \pi/4$. (Recall that $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$).

pts: /15

7. (a) Define the function $\ln x$ for $x > 0$.

   (b) If a, b are any positive real numbers and r is a rational number then

   * $\ln(ab) =$ _____
   * $\ln\left(\frac{a}{b}\right) =$ _____ $\ln\left(\frac{1}{b}\right) =$ _____
   * $\ln(a^r) =$ _____

   (c) Simplify the following expressions

   * $\ln \sec\theta + \ln cos\theta =$ _____
   * $\left(e^{\ln y - \ln x}\right)^{-1} + \frac{1}{y}\ln(e^x) =$ _____

pts: /15