MTH 133 - 12/08/98 THIRD MIDTERM	Fall 98 A. Corso - A. Minut	Name: ____________________ ____________________

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	10	
2	15	
3	15	
4	15	
5	15	
6	10	
7	10	
8	10	
Bonus	5	
TOTAL	100	

1. (a) Decide whether the following improper integral

$$\int_1^\infty \frac{\ln x}{x} dx$$

converge. If it does, find its value.

(b) Find the values of p for which

$$\int_1^2 \frac{dx}{x(\ln x)^p}$$

converges. $($*Hint*: consider the case $p = 1$ separately$)$.

pts: /10

2. Find the limits of the following sequences

$(a)\ a_n = (-1)^n \frac{\sin n}{n};$

$(b)\ a_n = \frac{9^n}{3^{2n+1}};$

$(c)\ a_n = \frac{1}{n} \int_1^n \frac{1}{x} dx.$

pts: /15

3. Determine if the following series converge. If they do, find their sum:

(a) $\sum_{n=2}^{\infty} \frac{\ln n}{n}$;

(b) $\sum_{n=2}^{\infty} \frac{\cos(n\pi)}{5^n}$;

(c) $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$.

pts: /15

4. Determine whether the following series converge or diverge. Give reasons for your answers.

(a) $\sum_{n=1}^{\infty} \frac{n}{n+1}$;

(b) $\sum_{n=1}^{\infty} \frac{n!}{(2n+1)!}$;

(c) $\sum_{n=1}^{\infty} 1+(-1)^n$.

pts: /15

5. Determine whether the following series converge absolutely, converge conditionally, or diverge. Give reasons for your answers.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^2+2n+1}$;

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n}}$.

pts: /15

6. (a) Find the interval of convergence of the following power series

$$\sum_{n=1}^{\infty} \sqrt[n]{n}(x-5)^n.$$

(b) Find the series' interval of convergence and, within this interval, the sum of the series as a function of x

$$\sum_{n=1}^{\infty} \frac{(x+1)^{2n}}{9^n}.$$

pts: /10

7. Find the MacLaurin series of

$$f(x) = \frac{x^3}{1+x}.$$

pts: /10

8. Estimate the error in the approximation

$$\sinh x = x + \frac{x^3}{3!}$$

when $|x| < 0.5$. (Hint: use R_4 not R_3).

pts: /10

Bonus. Find the length of the curve parametrized by the following equations

$$x(t) = e^t \cos t \qquad y(t) = e^t \sin t,$$

with $0 \leq t \leq \pi$.

pts: /5