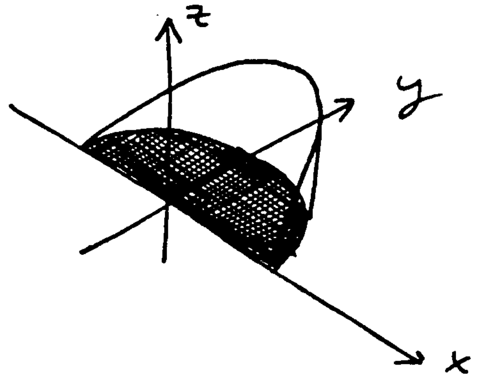
Math 213 — A. Corso

Practice for Exam 3

1. A wedge is cut from the cylinder $x^2+y^2=1$ by the plane $z=y$ and $z=0$, as shown.

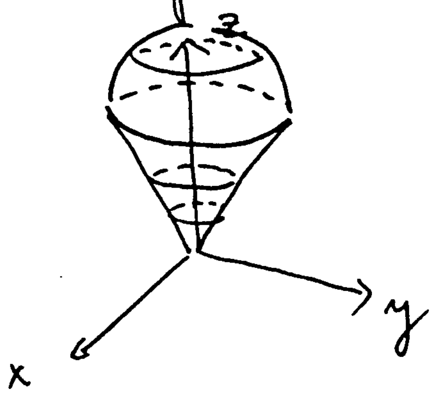
(a) Set up a triple integral in rectangular coordinates that represent the volume of the wedge



(b) Set up a triple integral in cylindrical coordinates that represents the volume of the wedge.

(c) Calculate the volume of the wedge by evaluating one of the integrals in (a) or (b).

2. Set up, but do not evaluate, a triple integral in spherical coordinates that represents the volume of the ice-cream cone shaped region bounded on top by the sphere of radius 2 and on the bottom by the cone $\varphi = \pi/6$.



3. Sketch the region of integration and evaluate the following integrals:

(a) $\int_0^{\pi} \left(\int_0^{\sin x} y \, dy \right) dx$

(b) $\int_0^{1} \left(\int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} \, dy \right) dx$

4. Reverse the order of integration in the following double integral. Do not compute the integral

$$\int_0^{\sqrt{\pi}} \left(\int_y^{\sqrt{\pi}} \cos x^2 \, dx \right) dy$$

5. (a) Set up a double integral to find the volume of the solid in the first octant bounded by the coordinates planes, the cylinder $x^2+y^2=4$ and by the plane $y+z=3$.

(b) Set up a triple integral to compute the same volume

(c) Compute this volume using either (a) or (b).

6. Let D be a thin plate of density δ, bounded by the parabola $x=y^2$, by the line $x+y=2$ and by the x-axis. Set up but do not evaluate:

(a) An integral for the mass;
(b) An integral for the area;
(c) An integral for the coordinates of the center of mass of D.

7. Let D be the smaller spherical cap cut from a solid ball of radius 2 by the plane $z=1$. Suppose the density of D is $\delta(x,y,z)=x^2+y^2$. Express the mass of D as an iterated triple integral in
(a) cartesian coordinates;
(b) cylindrical coordinates;
(c) spherical coordinates;
(d) change δ to cylindrical coordinates. Can you conclude anything about the x and y coordinates of the center of mass of D? How about the z coordinate?

8. Evaluate the integral $\int_{-1}^{0} \left(\int_{-1}^{y} e^{1-x^2}\, dx \right) dy$.

9. Write the integral $\int_{\sqrt{2}}^{2} \left(\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} x \, dy \right) dx$ in polar coordinates, and then evaluate it.

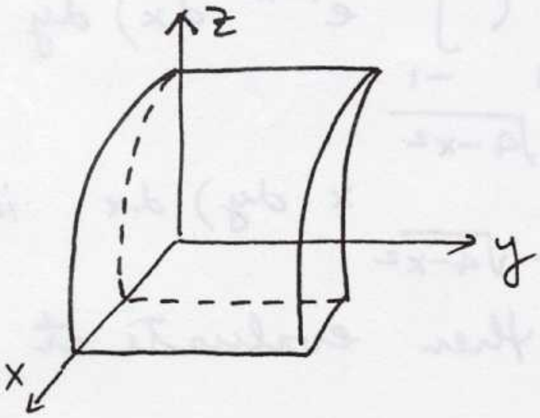
10. Consider the region in the first octant that is outside the cylinder $x^2+y^2=4$ and inside the sphere $x^2+y^2+z^2=9$. Set up (but do not evaluate) the integral expression in any form for the coordinates of the centroid.

11. Find the average value of $f(x,y)=x$ over the

"triangular" region in the plane bounded on the right by the graph of $y=x^2$, on the left by the graph of $x+y=2$ and from above by the graph of $y=4$.

12. Consider a bowl which is the lower hemisphere of the sphere $x^2+y^2+z^2=4$. Suppose this bowl is filled half-way to the top, so that at the surface of the water is in the plane $z=-1$. Find the volume of the water in the bowl.

13. Set up, do not evaluate, a triple iterated integral for the volume of the solid D that lies in the first octant between the cylinders $z=4-4x^2$ and $z=4-x^2$ with $0 \le y \le 3$.



14. Sketch the region of integration, write an equivalent double integral with the order of integration reversed and evaluate the integral

$$\int_0^2 \left(\int_0^{4-y^2} y \, dx \right) dy$$

15. Express as an iterated triple integral the volume of the region enclosed by the sphere $x^2+y^2+z^2=4$

and the cone $z = \sqrt{x^2+y^2}$:

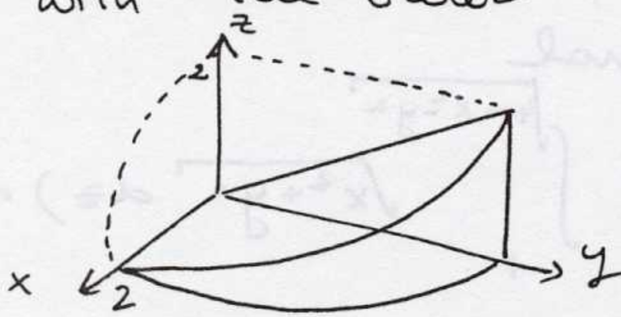
(a) using cylindrical coordinates ;

(b) using spherical coordinates .

16. Set up only !! Express as a triple iterated integral with limits the volume of the region in the cylinder $y = 4 - x^2$, above the xy-plane below the plane $2z = y$ and in the first octant :

(a) with the order $dz\,dx\,dy$;

(b) with the order $dy\,dx\,dz$.



17. Sketch and shade the region of integration of

$$\int_{-2}^{2} \left(\int_{-\sqrt{4-x^2}}^{0} \sin(x^2+y^2)\,dy \right) dx .$$

Change to polar coordinates and evaluate the integ

18. Consider the triple integral

$$\int_0^8 \left(\int_{\frac{1}{2}z}^{4} \left(\int_0^3 x^2\,dx \right) dy \right) dz .$$

(a) Sketch the region of integration .

(b) Rewrite the integral as an equivalent integral in the order $dz\,dy\,dx$.

(c) Find the exact value of the original integral.

19. Given the integral $\int_0^1 \left(\int_{\sqrt[3]{x}}^{1} e^{x/y}\, dy \right) dx$,

(a) sketch the region of integration;
(b) Evaluate the integral by first reversing the order of integration, then integrating.

20. Convert the integral
$$\int_{-\sqrt{3}}^{\sqrt{3}} \left(\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}} \left(\int_{1}^{\sqrt{4-x^2-y^2}} \sqrt{x^2+y^2}\, dz \right) dy \right) dx$$

(a) to cylindrical coordinates (do not evaluate);
(b) to spherical coordinates (do not evaluate)

21. Let $P = (\bar{x}, \bar{y})$ be the center of mass of a thin plate bounded by $y = x^2$ and $y = 8 - x^2$ and whose density is $\delta(x, y) = x^2$. Indicate which coordinate of P can be found without integration. Justify your answer. Also, set up (but do not integrate) the integrals for the other coordinate.

22. Convert the integral $\displaystyle\int_0^{2\pi}\left(\int_0^1\left(\int_0^{\sqrt{4-r^2}} r^2\,dz\right)dr\right)d\theta$

(a) to rectangular coordinates;
(b) to spherical coordinates.
Do not evaluate!