MA 223 PRACTICE QUESTIONS FOR THE FINAL 3/97

1. Which of the lines shown has slope $-1/3$?

A. B. C. D. E.

2. Suppose 280 tons of corn were harvested in 5 days and 940 tons in 20 days. If the relationship between tons T and days d is linear, express T as a function of d.

   A. $T = 5d + 280$ B. $T = -44d + 500$ C. $T = 44d + 60$ D. $T = 60d + 44$ E. None of these.

3. When 30 orange trees are planted per acre each tree yields 150 oranges For each additional tree per acre, the yield decreases by 3 oranges per tree. Express the total yield of oranges per acre, Y, as a function of the number of trees planted per acre, x, if $x \geq 30$.

   A. $Y = 4500 + 60x - 3x^2$ B. $Y = \frac{1}{3}x + 80$ C. $Y = 150x - 3x^2$ D. $Y = 240x - 3x^2$ E. None of these.

4. A manufacturer can sell dining-room tables for \$70 apiece. The manufacturer's total cost consists of a fixed overhead of \$8000 plus production costs of \$30 per table. How many tables must the manufacturer sell to break even?

   A. 80 B. 267 C. 200 D. 20 E. None of these.

5. If $f(u) = \sqrt{u+1}$ and $g(x) = x^2 + 7$ then $f(g(-1)) =$
   A. 0 B. 3 C. $\sqrt{7}$ D. 7 E. None of these.

6. If $f(x) = \dfrac{2}{x}$ then $\dfrac{f(x+\Delta x) - f(x)}{\Delta x} =$

   A. $-\dfrac{2}{x^2}$ B. $\dfrac{2}{x+\Delta x} - \dfrac{2}{x}$ C. $\dfrac{2}{x(x+\Delta x)}$ D. $-\dfrac{2}{x(x+\Delta x)}$ E. None of these.

7. The domain of $f(x) = \dfrac{1}{\sqrt[3]{x-1}}$ is all real numbers x such that

   A. $x \neq 1$ B. $x > 1$ C. $x > 0$ D. $x \neq 0$ E. None of these.

8. $\lim\limits_{x \to 1} \dfrac{x^2 + 4x - 5}{x^2 - 1} =$
   A. ∞ B. 0 C. 3 D. -3 E. None of these.

9. $\lim\limits_{x \to \infty} \dfrac{\ln x}{x} =$
   A. 0 B. 1 C. -1 D. ∞ E. None of these.

10. Suppose

$$f(x) = \begin{cases} Ax - 3 & \text{if } x < -1 \\ 3 - x + Ax^2 & \text{if } x \geq -1. \end{cases}$$

Find all values of the constant A so that the function $f(x)$ will be continuous at $x = -1$.
A. 1 B. 0 C. -1 D. $-\frac{7}{2}$ E. No value of A.

11. If the graph of $y = f(x)$ is

which of the following could be the graph of $y = f'(x)$?

A. B. C. D. E.

12. The derivative of $\dfrac{x^2+1}{x+5}$ is

A. $\dfrac{(x+5)2x-(x^2+1)}{(x+5)^2}$ B. $2x$ C. $\dfrac{(x+5)}{(x^2+1)^2}\cdot 2x$

D. $\dfrac{(x^2+1)+(x+5)2x}{(x+5)^2}$ E. $\dfrac{(x^2+1)-(x+5)2x}{(x+5)^2}$

13. If $y = (3-x^2)^3$ then $y'' =$

A. $-6x(3-x^2)^2$ B. $24x^2(3-x^2)-6(3-x^2)^2$ C. $6(3-x^2)$
D. $24x^2(3-x^2)$ E. None of these.

14. The line tangent to the graph of $f(x) = x - \dfrac{1}{x}$ at $x = 2$ has slope

A. 5/4 B. 3/4 C. 3/2 D. 0 E. None of these.

15. If the tangent line to the graph of $y = f(x)$ at $(2, 3)$ is shown in the diagram below, then $f'(2) =$

A. 3/2 B. 1 C. 2/3 D. -1 E. $-2/3$

16. After t years the population of a certain town is $P = 50 + 5t$ thousand people. A population P has an associated CO_2 level, $C(P) = (\sqrt{P^2+1})/2$. In 2 years (when $t = 2$), the rate at which $C(P)$ is changing with respect to t will be
A. $5/(2\sqrt{5})$ B. $150/\sqrt{3601}$ C. $30\sqrt{3601}$ D. $30/\sqrt{3601}$ E. None of these.

17. If $yx^2 + y^3 = x - y$. Then $y' =$
A. $1 - 2xy - 3y^2$ B. $1 - 2xy - x^2 - 3y^2$ C. $(1 - 2xy)/(3y^2 + 1)$ D. $(1 - 2xy)/(x^2 + 3y^2 + 1)$
E. None of these.

18. If $f''(x) > 0$ when $0 < x < 1$ and $2 < x < 3$ and $f''(x) \leq 0$ otherwise, then which of the following could be the graph of $y = f(x)$?

A. B. C. D. E.

19. If the concentration $C(t)$ of a certain drug remaining in the bloodstream t minutes after it is injected is given by $C(t) = t/(5t^2 + 125)$, then the concentration is a maximum when $t =$
A. 25 B. 15 C. 5 D. There is no maximum E. None of these.

20. If $f(x) = 2x^4 - 6x^2$ then which one of the following is true?
A. f has a relative max. at $x = \pm\sqrt{3/2}$ and a relative min at $x = 0$.
B. f has a relative max. at $x = 0$ and a relative min. at $x = \pm\sqrt{3/2}$.
C. f has a relative max. at $x = -\sqrt{3/2}$ and a relative min. at $x = \sqrt{3/2}$.
D. f has no relative max. points, but has relative min. at $x = \pm\sqrt{3/2}$.
E. None of these.

21. The derivative of a function f is $f'(x) = x^2 - \dfrac{8}{x}$. Then at $x = 2$, f has
A. an inflection point B. a relative maximum C. a vertical tangent
D. a discontinuity E. a relative minimum

22. If $f(x) = \frac{1}{3}x^3 - 9x + 2$, then on the closed interval $0 \leq x \leq 4$,
A. f has an absolute max. at $x = 3$ and an absolute min. at $x = 0$.
B. f has an absolute max. at $x = 4$ and an absolute min. at $x = 3$.
C. f has an absolute max. at $x = 0$ and an absolute min. at $x = 4$.
D. f has an absolute max. at $x = 0$ and an absolute min. at $x = 3$.
E. None of these.

23. The total cost in dollars to manufacture x units is given by the function $C = 3x^2 + x + 48$. For what value of x is the average cost a minimum?
A. 4 B. 0.17 C. There is no minimum D. 6.93 E. None of these.

24. A display case is in the shape of a rectangular box with a square base. Suppose the volume is 21 cubic ft and it costs \$1 per square ft. to build the glass top and \$0.50 per sq. ft. to build the sides and base. If x is the length of one side of the base, what value should x have to minimize the cost? Give your answer to two decimal places.
A. 3.04 ft. B. 2.41 ft. C. 3.74 ft. D. 2.24 ft. E. None of these.

25. Which of the following could be the graph of $y = \dfrac{1}{(x-1)(x+1)}$?

A. B. C. D. E.

26. The radius of a circular oil spill is increasing at the rate of 3 ft/min. How fast is the area increasing when the radius is 4 ft?
A. $24\pi \text{ft}^2/\text{min}$ B. $48\pi \text{ft}^2/\text{min}$ C. $8\pi \text{ft}^2/\text{min}$ D. $16\pi \text{ft}^2/\text{min}$ E. None of these

27. The level of air pollution in a certain city is proportional to the square of the population. Use differentials to estimate the percentage by which the air-pollution level will increase if the population increases by 5 percent.
A. 5% B. 10% C. 15% D. 20% E. 25%

28. Water is flowing into a tank which is in the shape of a right circular cylinder standing on its circular base. If the water is flowing in at a rate of 80 cu. ft. per min. and the radius of the base of the tank is 4 ft., how fast is the water rising when the water is 10 ft. deep?
A. $\frac{\pi}{5}$ ft/min B. 5π ft/min C. $\frac{50}{\pi}$ ft/min D. $\frac{5}{\pi}$ ft/min E. None of these.

29. Each machine at a certain factory can produce 50 units per hour. The setup cost is \$80 per machine and the total operation costs are \$5 per hour. How many machines should be used to produce 8000 units at the least possible cost? (Give your answer to the nearest integer.)
A. 2 B. 3 C. 5 D. 6 E. None of these.

30. A population grows exponentially ($Q = Q_0 e^{kt}$). In 1960 it was 50,000 and in 1965 it was 100,000. What was the population in 1970?
A. 200,000 B. 150,000 C. 250,000 D. 300,000 E. 225,000

31. If $18^x = \sqrt{3}$, then in which of the following intervals does x lie?
A. $(0, 1)$ B. $(-1, 0)$ C. $(1, 2)$ D. $(-2, -1)$ E. None of these.

32. If $y = \ln\sqrt{1-x^2}$ then $\dfrac{dy}{dx} =$
A. $\dfrac{1}{\sqrt{1-x^2}}$ B. $\dfrac{-2x}{\sqrt{1-x^2}}$ C. $\dfrac{-x}{1-x^2}$ D. $\dfrac{1}{2(1-x^2)}$ E. None of these.

33. If $y = e^{x^2}$ then $\dfrac{dy}{dx} =$

A. e^{x^2} B. $x^2 e^{x^2-1}$ C. $2xe^{x^2-1}$ D. $2xe^{x^2}$ E. None of these.

34. If $e^{x+y} = xy + e$ find y' at the point $(0, 1)$.

A. $1/e$ B. $(1-e)/e$ C. $(2-e)/e$ D. -1 E. None of these.

35. What lump sum of money should be deposited in a money market certificate paying 8.25% interest compounded monthly to amount to 5000 in 10 years? Give your answer to the nearest dollar. $(B(t) = P(1 + r/k)^{kt})$.

A. \$2197 B. \$4669 C. \$2740 D. \$2262 E. None of these

36. How quickly will money double if it is invested at a rate of 8 percent compounded continuously? Give your answer to two decimal places. $(B(t) = Pe^{rt})$

A. 0.87 years B. 25 years C. 5.55 years D. 8.66 years E. None of these.

37. The graph of $y = xe^{-x}$ looks most like which of the following?

A. B. C. D. E.

38. Suppose the total cost in dollars of producing q units is $C(q) = 2e^{-q} + 3q^2 - 2$. Calculate the marginal cost, MC, when 5 units have been produced and calculate the actual cost, AC, of producing the 6th unit. Give your answer to the nearest cent.

A. $MC = \$29.99, AC = \$32.99$ B. $MC = AC = \$29.99$
C. $MC = \$29.99, AC = \$36.00$ D. $MC = \$30.01, AC = \$32.99$
E. $MC = AC = \$30.01$

39. At a certain factory, the daily output is $Q(K) = 4000K^{1/2}$ units, where K denotes the firm's capital investment. Use differentials to estimate the percentage increase in output that will result from a 1 percent increase in capital investment.

A. 1% B. 1.5% C. 0.5% D. 2% E. None of these.

40. A cylindrical can with no top has been made from 27π square inches of metal. Express the volume, V, of the can as a function of its radius, r.

A. $V = 27\pi r^2$ B. $V = \frac{\pi}{2} r(27 - r^2)$ C. $V = \pi r^2(27 - r^2 - 2r)$
D. $V = 27\pi^2 r^2$ E. $V = \frac{4}{3}\pi r^2(27 - r^2)$

41. In 1984, the rate for interstate telegrams was \$8.45 for 10 words or less plus 45 cents for each additional word. Express the cost, C, of sending a telegram as a function of its length, x.

A. $C = 845x + 45$ B. $C = 845 + 45x$ C. $C = \begin{cases} 845x, & \text{if } 0 \leq x \leq 10, \\ 845x + 45 & \text{if } x > 10. \end{cases}$

D. $C = \begin{cases} 845, & \text{if } 0 \leq x \leq 10, \\ 845x + 45, & \text{if } x > 10. \end{cases}$ E. $C = \begin{cases} 845, & \text{if } 0 \leq x \leq 10 \\ 845 + 45(x - 10), & \text{if } x > 10. \end{cases}$

SOLUTION - MA 223 PRACTICE EXAM

1. A; 2. C; 3. D; 4. C; 5. B; 6. D; 7. A; 8. C; 9. A; 10. D; 11. E; 12. A; 13. B; 14. A; 15. B; 16. B; 17. D; 18. C; 19. C; 20. B; 21. E; 22. D; 23. A; 24. B; 25. E; 26. A; 27. B; 28. D; 29. B; 30. A; 31. A; 32. C; 33. D; 34. B; 35. A; 36. D; 37. B; 38. A; 39. C; 40. B; 41. E.