

## Business Calculus (Mathematics 223 – Fall 1997)

**Exercise #1:** Given  $f(x) = \frac{1}{\sqrt{x+3}} + x + 2$ : (a) Find the domain of  $f(x)$ .  
(b) Find  $h(x)$  and  $g(u)$  such that  $f(x) = g(h(x))$ .

**Problem #2:** Given  $f(x) = \frac{x}{x-2} + 1$ : (a) Find  $x$  and  $y$  intercepts, if any.  
(b) Point out the  $x$  value(s) at which  $f(x)$  is discontinuous.

**Exercise #3:** Find the equation of the line passing (1,2) and perpendicular to  $2x = 3x + 1$ .

**Exercise #4:** A manufacturer buys \$20,000 worth of machinery that depreciates at a constant rate so that its value after 10 years will be \$12,000. (a) Express the value of the machinery as a function of its age.  
(b) Compute the value of the machinery at the end of the 4th year.

**Exercise #5:** A closed box with a square base has a surface area of 4,000 square centimeters. Express its volume as a function of the length of its base.

**Problem #6:** A manufacturer can sell its product for \$90 apiece. Its total cost consists of a fixed cost of \$8,000 and production cost of \$50 apiece. How many products must the manufacturer sell to make a profit of \$200?

**Exercise #7:** Find the following limits: (a)  $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-4}$  (a)  $\lim_{x \rightarrow 0} \frac{x^2-x}{x}$ .

**Problem #8:**  $f(x) = \begin{cases} x+9 & x < 2 \\ x^2 + 4A - 1 & x \geq 2 \end{cases}$   
Find the value of  $A$  so that  $f(x)$  is continuous at  $x = 2$ .

**Problem #9:** Find the following limits: (a)  $\lim_{x \rightarrow \infty} \frac{3x^2 - 6x + 2}{2x^2 - 9}$  (b)  $\lim_{x \rightarrow 0^-} \left(1 + \frac{x+2}{x}\right)$ .

**Problem #10:** The graph of  $y = f(x)$  is as shown, find the following:

- (a) All  $x$  values at which the limit of  $f(x)$  does not exist.
- (b) All  $x$  values at which  $f(x)$  is not continuous.
- (c) All vertical and horizontal asymptotes, if any.

**Problem #11:** Let  $f(x) = x^2 - 6x + 9$  and  $g(x) = 2x - 3$ . Find  $f(g(x))$  and  $g(g(x))$ . Simplify your answer as much as possible.

**Problem #12:** Find the equation of the line passing through the point  $(-2, 3)$  and parallel to the line  $6x - 3y = 5$ . Write the equation in slope-intercept form.

**Exercise #13:** Compute the following limits (if they exist):

$$\begin{array}{ll} (a) \lim_{x \rightarrow 3} \frac{x^2 + 5x + 6}{x^2 - 4} & (b) \lim_{x \rightarrow 2} \frac{x^2 + 5x + 6}{x^2 - 4} \\ (c) \lim_{x \rightarrow -2} \frac{x^2 + 5x + 6}{x^2 - 4} & (d) \lim_{x \rightarrow +\infty} \frac{x^2 + 5x + 6}{x^2 - 4}. \end{array}$$

**Exercise #14:** What is the domain of the function  $f(x) = \frac{\sqrt{2x-6}}{x^2-16}$ ?

**Exercise #15:** Consider the function  $f$  with the following graph:

True or False:

- |   |   |   |
|---|---|---|
| (a) $f$ is defined at $x = 5$             | T | F |
| (b) $\lim_{x \rightarrow 5} f(x)$ exists  | T | F |
| (c) $\lim_{x \rightarrow -3} f(x)$ exists | T | F |
| (d) $f$ is continuous at $x = -3$         | T | F |

**Exercise #16:** Let  $f(x) = \begin{cases} x^2 - 4x + 4 & \text{if } x < 3 \\ 13 & \text{if } x = 3 \\ 2x - 5 & \text{if } x \geq 3 \end{cases}$

- (a) Compute  $\lim_{x \rightarrow 3^-} f(x)$ .  
 (b) Compute  $\lim_{x \rightarrow 3^+} f(x)$ .  
 (c) Does  $\lim_{x \rightarrow 3} f(x)$  exist? If so, what is its value? If not, explain why not.  
 (c) Is  $f$  continuous at  $x = 3$ ? Why or why not?

**Exercise #17:** An open box with a square base is to be constructed for \$72. The sides of the box cost \$4 per square meter. The base costs \$7 per square meter. Express the volume of the box as a function of the length of its base.

**Exercise #18:** A bookstore can obtain a certain gift book from the publisher at a cost of \$3 per book. The bookstore has been offering the book at the price of \$15 per copy and, at this price, has been selling 200 copies a month. The bookstore estimates that for each \$1 reduction in the price, 20 more books will be sold each month.

- (a) Express the number  $N$  of books sold per month as a function of the selling price  $x$ .  
 (b) Express the monthly profit  $P$  as a function of the selling price  $x$ .  
 (c) Sketch the graph of  $P(x)$ . Be sure to label all intercepts and *important points*.  
 (d) What is the optimal selling price?