MATH 251 Midterm # 2	Spring 96–Sec. 01-02-03 Corso – Higgins	Name: ______________________ ______________________

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	20	
2	20	
3	20	
4	20	
5	20	

WHERE DO YOU SIT?

J														
I														
H														
G														
F														
E														
D														
C														
B														
A														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

BLACKBOARD

total: /100

1. (a) Compute, using the chain rule,

$$\frac{\partial z}{\partial x} \qquad \frac{\partial z}{\partial y}$$

if $z = x + f(u)$ and $u = xy$. Then, show that

$$x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y} = x.$$

(b) Find the derivative of

$$f(x, y) = x^2 + y^2$$

in the direction of the unit tangent vector (i.e., $\widehat{T} = \vec{v}/|\vec{v}|$) of the curve

$$\vec{r}(t) = (\cos t + t\sin t)\widehat{\imath} + (\sin t - t\cos t)\widehat{\jmath} \qquad t > 0.$$

$\boxed{\quad /20}$

2. Let us consider the function

$$f(x,y) = xy$$

defined on the closed disk $R = \{(x,y) \mid x^2 + y^2 \leq 10\}$.

(*a*) Find the unique critical point of f in the interior of the disk R. What is its nature?

(*b*) Find the extreme values of $f(x,y)$ on the boundary of R by using

1. the method of Lagrange multipliers
(note that the constraint is given by $g(x,y) = x^2 + y^2 - 10 = 0$);
2. the parametrization of the boundary of R given by

$$x = \sqrt{10}\cos\theta \qquad y = \sqrt{10}\sin\theta \qquad 0 \leq \theta \leq 2\pi.$$

(*c*) What is the absolute maximum and the absolute minimum of $f(x,y)$ on R? Where are they attained?

(*d*) Select the graph of $f(x,y)$: possibilities $A - D$.

(*e*) Describe the level curves of $f(x,y)$? Mark the points where ∇f is a scalar multiple of ∇g.

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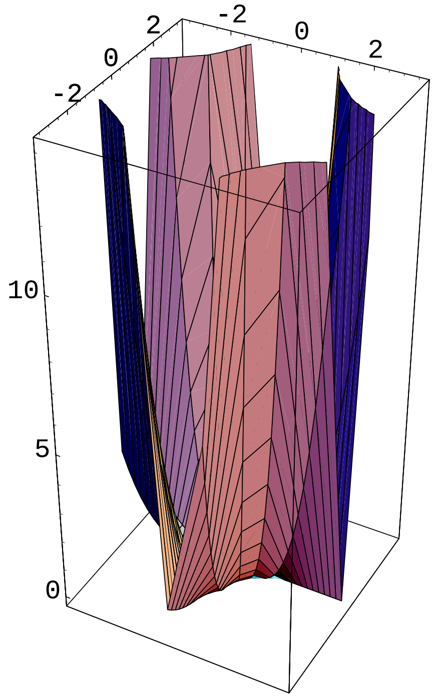
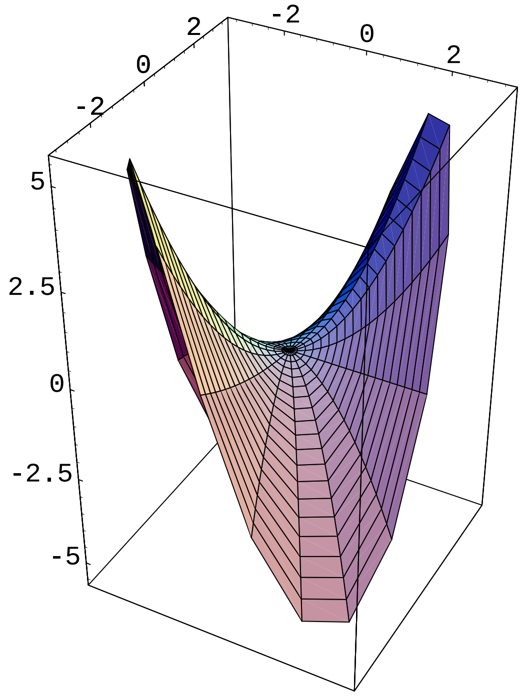
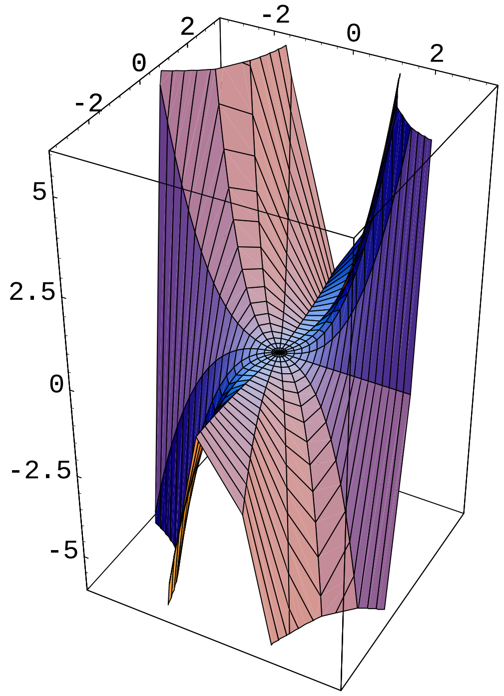
A



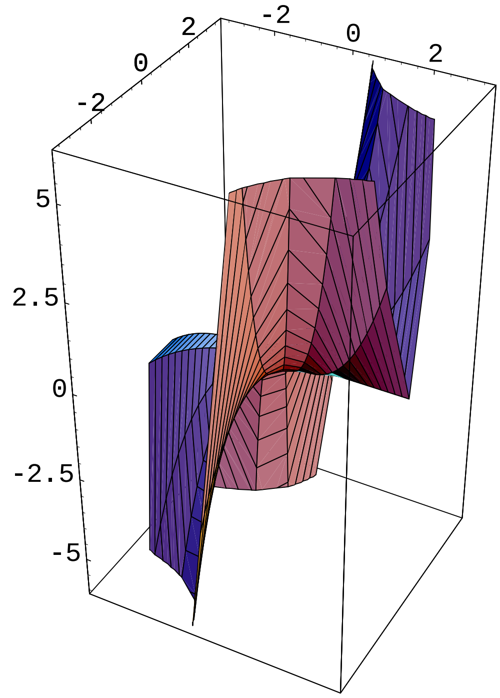
B



C



D



3. Let R be the triangle bounded by the x-axis, by $x = 3$ and by the line $y = x$.

(a) Using cartesian coordinates, find the average value over R of $f(x, y) = y^3$.

(b) Using polar coordinates, find the average value over R of $g(r, \theta) = \sin \theta$.

/20

4. (a) Show that the differentable functions

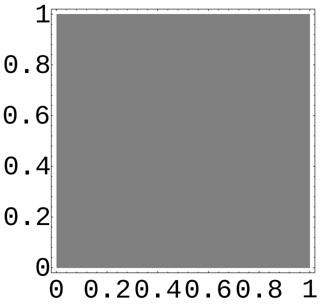
$$x = u^2 - v^2 \qquad y = 2uv$$

transform one-to-one the unit square G into the region R (Hint: see what happens to the boundary of the square).

(b) Evaluate the double integral

$$\iint_R y\, dx dy.$$

$/20$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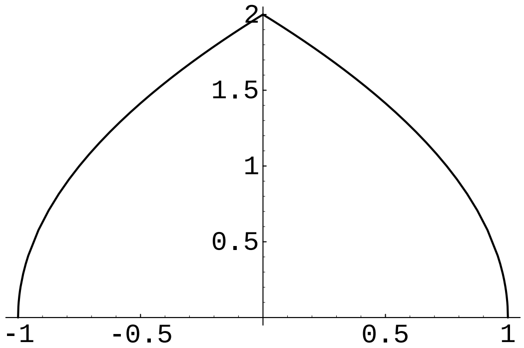
0.6
0.4
0.2
0
0 0.20.40.60.8 1


2
1.5
1
0.5
-1
-0.5
0.5
1

5. Let D be the solid of constant density $\delta(x, y, z) = 1$ bounded below by the paraboloid $z = x^2 + y^2$ and bounded above by the plane $z = 9$. Find the centroid of the solid D. (Hint: use cylindrical coordinates).

Recall that

$$\overline{x} = \frac{M_{yz}}{M} \qquad \overline{y} = \frac{M_{xz}}{M} \qquad \overline{z} = \frac{M_{xy}}{M}$$

$$M = \iiint_D \delta(x, y, z) dV$$

$$M_{yz} = \iiint_D x\delta(x, y, z) dV \quad M_{xz} = \iiint_D y\delta(x, y, z) dV \quad M_{xy} = \iiint_D z\delta(x, y, z) dV$$

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