MATH 251 Midterm # 1	Spring 96–Sec. 01-02-03 Corso – Higgins	Name: _____ _____

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	15	
2	10	
3	15	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	

total: /100

1. (a) The planes

$$x + 2y + z = 1 \quad \text{and} \quad x - y + 2z = -8$$

intersect in a line. Find the parametric equations for the line.

(b) Find the distance from the origin to the line found in part (a).

(c) Find the distance from the origin to the line in the xy-plane given by the equation

$$3y + x - 6 = 0.$$

(d) To compute the distance from a point to a line one uses two different formulas: one of them involves the dot product while the other involves the cross product. Why?

2. The helix

$$\vec{r}_1(t) = \cos t\hat{i} + \sin t\hat{j} + t\hat{k}$$

intersect the curve

$$\vec{r}_2(t) = (1+t)\hat{i} + t^2\hat{j} + t^3\hat{k}$$

at the point $(1, 0, 0)$. Find the angle of intersection of these curves.

/10

3. Consider the curve

$$\vec{r}(t) = t\widehat{i} + \ln(\cos t)\widehat{j} + 2\widehat{k} \qquad -\pi/2 < t < \pi/2.$$

Find $\widehat{T}$, $\widehat{N}$, $\widehat{B}$, κ and τ.

4. (a) Describe the set of points in the first octant whose cylindrical coordinates satisfy

$$r \leq 4 \qquad 0 \leq \tan\theta \leq 1 \qquad 0 \leq z \leq 4;$$

(b) Describe the set of points in the space whose cartesian coordinates satisfy

$$0 \leq z \leq x^2 + 4y^2.$$

/10

5. Match each of the following functions

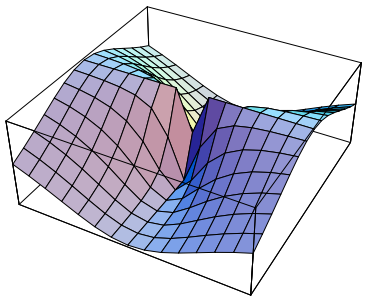
$$f(x,y)=\frac{x^2-y^2}{x^2+y^2} \qquad g(x,y)=\sin\left(\sqrt{x^2+y^2}\right) \qquad h(x,y)=\cos(xy) \qquad k(x,y)=\frac{1}{4x^2+y^2}$$

(a) with its graph (labeled A-D);

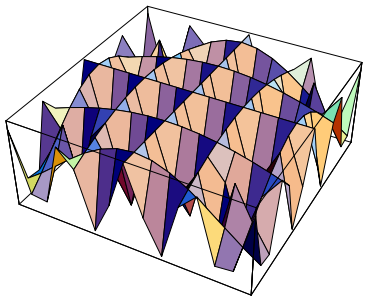
(b) with its level curves (labeled I-IV).

Give reasons for your choice.

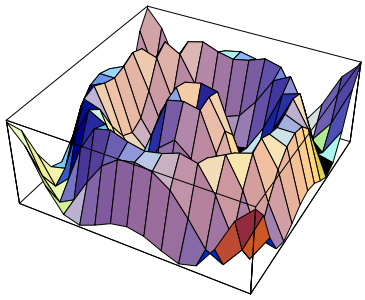
A



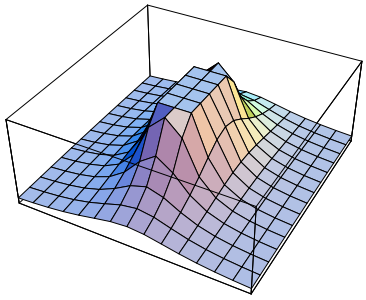
B



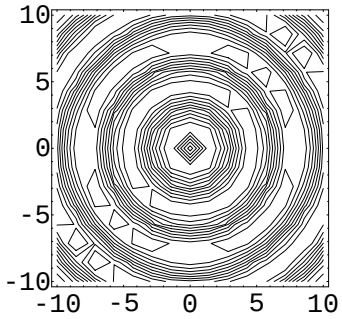
C



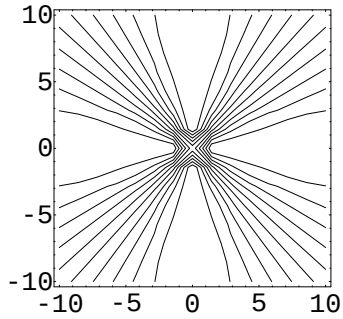
D



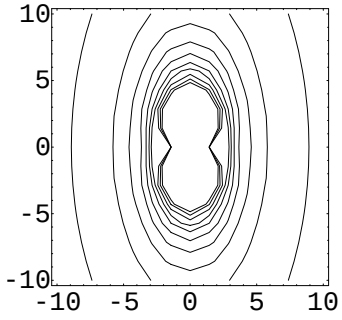
I



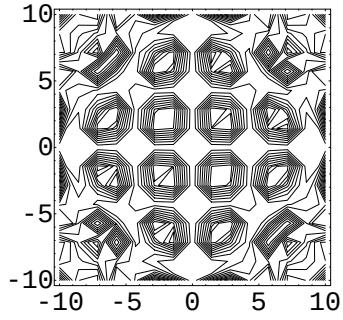
II



III



IV



6. Find the domain and the range of the function

$$f(x, y, z) = e^{-(x^2+y^2+z^2)}.$$

Describe the level surfaces of $f(x, y, z)$ and sketch some of them.

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7. Let $f(x,y) = x^3 \ln(x-y)$. Find

$$\frac{\partial f}{\partial x}, \quad \frac{\partial f}{\partial y}, \quad \frac{\partial^2 f}{\partial x \partial y}, \quad \frac{\partial^2 f}{\partial y \partial x}.$$

/10

8. (a) Let

$$f(x,y) = \frac{x^2 - y^2}{x^2 + y^2}.$$

Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin?

(b) Let

$$f(r,\theta) = e^r \frac{\sin(r\theta)}{r\theta}.$$

Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin?

9. Two legs of a right triangle are measured as 5m and 12m respectively, with a possible error in measurement of at most 0.2cm in each. Use differentials to estimate the maximum error in the calculated value of

   (a) the area of the triangle;

   (b) the length of the hypotenuse.

10. Let

$$z = F(x, y) = x^2 y^2,$$

where $x = r^2 - s^2$ and $y = 2rs$. Use the chain rule to find

$$\frac{\partial z}{\partial r} \qquad \frac{\partial z}{\partial s} \qquad \frac{\partial^2 z}{\partial r \partial s}.$$

Evaluate the previous partial derivatives at $s = 1$ and $r = 1$.