Math 265 - Dr. A. Corso PRACTICE EXAM	No calculators or cheat sheets are allowed during the test.	The exam will be on Wednesday April 16 from 7-9 pm in MTHW 210

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1. Let V be a vector space with basis $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$. Show that

$$\mathbf{u}_1 = \mathbf{v}_1, \qquad \mathbf{u}_2 = \mathbf{v}_1 + \mathbf{v}_2, \qquad \mathbf{u}_3 = \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$$

is also a basis for V. What is the dimension of V?

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2. Mark each of the following True or False:

True ☐	False ☐	Every generating subset of V is a basis for V.
True ☐	False ☐	Every independent subset of V is a basis for V.
True ☐	False ☐	Every independent set of vectors in V is a basis for the vectors span.
True ☐	False ☐	If each vectors in V is a unique linear combination of the vectors in the set $\{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$ then this set consists of linearly independent vectors.
True ☐	False ☐	Let B be an n by n matrix. If the nullspace of B is $\mathbf{0}$ then the equation $B\mathbf{X} = \mathbf{b}$ has at least one solution.
True ☐	False ☐	Let B be an n by n matrix. If the columns of B are linearly independent then the equation $B\mathbf{X} = \mathbf{b}$ has infinitely many solutions.

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3. Find a basis for the range and for the nullspace of A if

$$A = \begin{bmatrix} 0 & 1 & 1 & -1 \\ 1 & -1 & 1 & 2 \\ 2 & 2 & 1 & -1 \end{bmatrix}.$$

Verify the rank nullity theorem in this specific case.

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4. Let V be the vector space of all continuous real valued functions defined on the interval $[0,1]$ equipped with the inner product

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt$$

for all $f, g \in V$. Compute the angle between the following two functions

$$x(t) = t \qquad y(t) = t^2 - t + 1.$$

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5. The vectors $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ in $\mathbf{R}^n$ satisfy the following conditions

$$\| \mathbf{u} \| = 1 \qquad \| \mathbf{v} \| = 2 \qquad \| \mathbf{w} \| = 3 \qquad \langle \mathbf{u}, \mathbf{v} \rangle = -1 \qquad \langle \mathbf{u}, \mathbf{w} \rangle = 2 \qquad \langle \mathbf{w}, \mathbf{v} \rangle = 0.$$

Find the length of the vector $\mathbf{u} - 3\mathbf{v} + 2\mathbf{w}$.

6. The vectors $\mathbf{w}_1 = (1, 1, -1, 1)$, $\mathbf{w}_2 = (0, 1, 0, 1)$, and $\mathbf{w}_3 = (2, 1, 2, -1)$ form a basis for the subspace W.

   (a) Find an orthonormal basis $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ for the subspace W.

   (b) Find an orthonormal basis vector $\mathbf{v}$ for $W^{\perp}$.

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7. Factor the following polynomials

$$\lambda^3 - \lambda^2 - 14\lambda + 24;$$
$$\lambda^3 - 9\lambda^2 + 24\lambda - 16;$$
$$\lambda^4 + 5\lambda^3 + 9\lambda^2 + 7\lambda + 2.$$

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8. Find the eigenvalues and the eigenvectors of the matrix A if

$$A = \begin{bmatrix} 2 & 3 \\ -3 & 2 \end{bmatrix}.$$

Make sure to check that the vectors you obtained are indeed eigenvectors.

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9. Find the eigenvalues and the eigenvectors of the matrix A if

$$A = \begin{bmatrix} 1 & -3 & 3 \\ 0 & -5 & 6 \\ 0 & -3 & 4 \end{bmatrix}.$$

Make sure to check that the vectors you obtained are indeed eigenvectors.

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10. Solve the following initial value problem

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}.$$

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11. Let A and B be two n by n invertible matrices. Show that AB and BA have the same eigenvalues.