MATH 265 - 2/25/97 FIRST MIDTERM	Spring 97–Sec. 01-03 Dr. A. Corso	Name: ____________________________ ____________________________

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; CIRCLE YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	15	
2	15	
3	15	
4	10	
5	10	
6	10	
7	15	
8	10	
E.C.	10	
TOTAL	100	

WHERE DO YOU SIT?

J														
I														
H														
G														
F														
E														
D														
C														
B														
A														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14

BLACKBOARD

1. Let A and B be the following 3×3 matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 2 & -1 \\ 0 & 2 & 1 \end{bmatrix}.$$

- Find the matrix $C = AB - BA$.
- Find the determinant of the matrix C.
- How can you write the transpose C^t of C in terms of A^t and B^t?

grade: /15

2. Consider the non homogeneous system of linear equations $A\mathbf{X} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 & 1 \\ -1 & -2 & 1 & 1 & 0 \\ 1 & 2 & -3 & -7 & -2 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -8 \end{bmatrix}.$$

- Find all the solutions of the non homogeneous system $A\mathbf{X} = \mathbf{b}$.
- Find all the solutions of the homogeneous system $A\mathbf{X} = \mathbf{0}$.
- Suppose that $\mathbf{X_1} = (1, 2, 3, 4, 5)$ is a particular solution of $A\mathbf{X} = (18, 2, -42)$. Find all the solutions of the system $A\mathbf{X} = (18, 2, -42)$.

grade: /15

3. Suppose that you have to solve the homogeneous system $A\mathbf{X} = \mathbf{0}$ with A a 4×5 matrix given by

$$A = \begin{bmatrix} 1 & -1 & 0 & 1 & -1 \\ -1 & 2 & -1 & -1 & 1 \\ 2 & -3 & 3 & 1 & -2 \\ 1 & 0 & -3 & 2 & -1 \end{bmatrix}.$$

Find

- all the solutions of the homogeneous system in terms of a basis of the nullspace of A;
- a basis for the range of A (make sure to write the column vectors that you leave out as a linear combination of the basis vectors that you chose)

if you know that

$$\texttt{rref}(A|\mathbf{0}) = \left[\begin{array}{ccccc|c} 1 & 0 & 0 & 1/2 & -1 & 0 \\ 0 & 1 & 0 & -1/2 & 0 & 0 \\ 0 & 0 & 1 & -1/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

grade: /15

4. Solve the non homogeneous system of linear equations

$$\left\{ \begin{array}{rrcl} x & +\frac{1}{2}\,y & = & 1 \\ \frac{1}{2}\,x & +\frac{1}{3}\,y & = & 1 \end{array} \right.$$

- by using Cramer's rule.
- by writing in in the form $A\mathbf{X} =$ and then solving it using $\mathbf{X} = A^{-1}\mathbf{b}$.

grade: /10

5. Let B be the 3×3 matrix

$$B = \begin{bmatrix} 1 & -2 & 3 \\ 2 & k & 6 \\ -1 & 3 & k-3 \end{bmatrix}$$

where k is an arbitrary number. Find for which values of k does one have that $\det(B) = 0$?

grade: /10

6. Let V be the vector space consisting of all 2×2 matrices with real entries: $V = M_{2\times 2}(\mathbb{R})$. Show that the set W consisting of all 2×2 matrices with *trace*[1] zero, i.e.

$$W = \left\{ \begin{bmatrix} a & b \\ c & -a \end{bmatrix} : a, b, c \text{ any real number} \right\}$$

is a subspace of V.

Can you find a basis for W? How many elements does it have?

grade: /10

[1] The trace of a square matrix is the sum of the elements on the main diagonal.

7. Show that the vectors $\mathbf{u} = (0, 1, -1, 1)$, $\mathbf{v} = (1, 0, -2, 1)$, and $\mathbf{w} = (1, 1, 0, 2)$ are linearly independent vectors in $\mathbf{R}^4$.

   Let W be the subspace spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$: i.e., $W = \text{span}\langle \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle$.

   Show that the vector $\mathbf{x} = (-2, 6, 10, 4)$ is in W and find α, β, and γ such that

$$\mathbf{x} = \alpha\mathbf{u} + \beta\mathbf{v} + \gamma\mathbf{w}.$$

   Can you find a fourth vector $\mathbf{y}$ su that

$$\mathbf{R}^4 = \text{span}\langle \mathbf{u}, \mathbf{v}, \mathbf{w}, \mathbf{y} \rangle?$$

   Give such a $\mathbf{y}$.

grade: /15

8. (a) Find a basis for the plane in $\mathbf{R}^3$ given by the equation $x+y=0$ (Note that this means: $1\cdot x+1\cdot y+0\cdot z = 0$).

(b) Given the vectors $\mathbf{u} = (1,2,0)$ and $\mathbf{v} = (-1,0,2)$, find an equation for the plane that they span.

(c) Give a precise description of the line of intersection of the planes given by the equations in part (a) and (b): i.e., give a basis vector for that line.

grade: /10

Extra Credit If you have time left, choose one of the following problems:

(a) Let A be the following 2×2 matrix

$$A = \begin{bmatrix} 3 & -\frac{1}{2} \\ 2 & 1 \end{bmatrix}$$

For which value of λ does one have that

$$A\mathbf{v} = \lambda \mathbf{v}$$

for some vector $\mathbf{v} = (v_1, v_2)$?

(b) Let A and B be two $n \times n$ invertible matrices. Show that $I_n - AB$ is invertible if and only if $I_n - BA$ is invertible. (Hint: $B(I_n - AB) = (I_n - BA)B$)

(c) Let V be the vector space of all functions from $\mathbf{R}$ to $\mathbf{R}$. Let V_e be the subset of the even functions (i.e. functions such that $f(-x) = f(x)$ for all x); let V_o be the subset of the odd functions (i.e. functions such that $f(-x) = -f(x)$ for all x).

* Show that V_e and V_o are subspaces of V.
* Show that every function can be written as the sum of an even function plus an odd function (e.g. $e^x = (e^x + e^{-x})/2 + (e^x - e^x)/2 = \cosh(x) + \sinh(x)$).

grade: /10