Math 265 - Dr. A. Corso PRACTICE EXAM	No calculators or cheat sheets are allowed during the test.	The exam will be on Tuesday February 25 from 7-9 pm in SMTH 108

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1. Let A be the following 3×3 matrix

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & 2 \\ 2 & -1 & 1 \end{bmatrix}.$$

   - Find the inverse A^{-1} of A.
   - Find the transpose A^t of A and the inverse $(A^t)^{-1}$ of A^t.
   - Find $\det(A)$, $\det(A^{-1})$, $\det(A^t)$, and $\det((A^t)^{-1})$.

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2. Consider the non homogeneous system of linear equations $A\mathbf{X} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 1 & -1 & -1 & 4 \\ 2 & 1 & 1 & 0 & 3 \\ -1 & 3 & 1 & 5 & 4 \end{bmatrix} \qquad \mathbf{X} = \begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 5 \\ -1 \\ 3 \end{bmatrix}.$$

   - Find all the solutions of the non homogeneous system $A\mathbf{X} = \mathbf{b}$.
   - Find all the solutions of the homogeneous system $A\mathbf{X} = \mathbf{0}$.
   - Suppose that $\mathbf{X_1} = (2, -1, 0, 1, 3)$ is a particular solution of $A\mathbf{X} = (12, 12, 12)$. Find all the solutions of the system $A\mathbf{X} = (12, 12, 12)$.

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3. Suppose that you have to solve the homogeneous system $A\mathbf{X} = \mathbf{0}$ with A a 4×5 matrix given by

$$A = \begin{bmatrix} 1 & 1 & -1 & 0 & 1 \\ -1 & -1 & 2 & -1 & 1 \\ 2 & 2 & -3 & 3 & 1 \\ 1 & 1 & 0 & -3 & 2 \end{bmatrix}.$$

   Find

   - all the solutions of the homogeneous system in terms of a basis of the nullspace of A;
   - a basis for the range of A

   if you know that

$$\texttt{rref}(A|\mathbf{0}) = \left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 7/2 & 0 \\ 0 & 0 & 1 & 0 & 5/2 & 0 \\ 0 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

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4. Solve the non homogeneous system of linear equations

$$\begin{cases} 2x & & +3z & = & 1 \\ & -y & +z & = & -4 \\ -x & +3y & +3z & = & 4 \end{cases}.$$

   by using Cramer's rule.

5. Find a basis for the solution space of the homogeneous system of linear equations

$$\begin{cases} x & +y & -z & +2w & = & 0 \\ -x & +y & +4z & -3w & = & 0 \\ & 2y & +5z & w & = & 0 \end{cases}.$$

Check that $\mathbf{X_1} = (-5, 2, -1, 1)$ is a solution and write it in terms of that basis.

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6. Let B be the 3×3 matrix

$$B = \begin{bmatrix} 1 & k & 1 \\ k & 1 & 1 \\ 1 & (2k+1) & (k+2) \end{bmatrix}$$

where k is an arbitrary number. Find for which values of k one has that $\det(B) = 0$. Find for which values of k one has that $\det(B) = 3/8$.

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7. Let V be the vector space consisting of all 2×2 matrices with real entries: $V = M_{2\times 2}(\mathbb{R})$. Show that the set W consisting of all 2×2 symmetric matrices

$$W = \left\{ \begin{bmatrix} a & b \\ b & c \end{bmatrix} \ : a, b, c \text{ any real number} \right\}$$

is a subspace of V.

EXTRA CREDIT: Can you find a basis for V? Can you find a basis for W? How many elements do they have?

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8. Consider the following subset of $\mathbb{R}^5$

$$Z = \left\{ \begin{pmatrix} t \\ 3s + 4t - 2r \\ s \\ r \\ 3s - r + t \end{pmatrix} \ : \ s, t, r \text{ any real number} \right\}.$$

- Show that Z is closed under addition and under scalar multiplication: hence it is a subspace of $\mathbb{R}^5$. Can you exhibit a basis for Z?
- Show that Z is also the nullspace of some 2×5 matrix A. Find A.

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9. Show that the vectors $\mathbf{u} = (1, -1, 1, 0)$, $\mathbf{v} = (1, -2, 0, 1)$, and $\mathbf{w} = (2, 0, 1, 1)$ are linearly independent vectors in $\mathbb{R}^4$.

   Let W be the subspace spanned by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$: i.e., $W = \mathrm{span}\langle \mathbf{u}, \mathbf{v}, \mathbf{w} \rangle$.

   Show that the vector $\mathbf{x} = (27, 10, 22, 5)$ is in W and find α, β, and γ such that

$$\mathbf{x} = \alpha \mathbf{u} + \beta \mathbf{v} + \gamma \mathbf{w}.$$

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10. Find the value for λ such that

$$\det \begin{bmatrix} 2-\lambda & 1 & 0 \\ -1 & -\lambda & 1 \\ 1 & 3 & 1-\lambda \end{bmatrix} = 0.$$
