

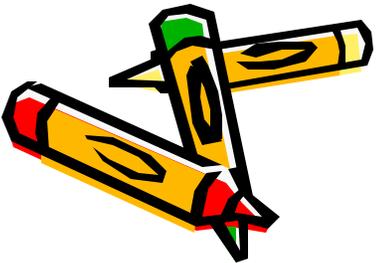
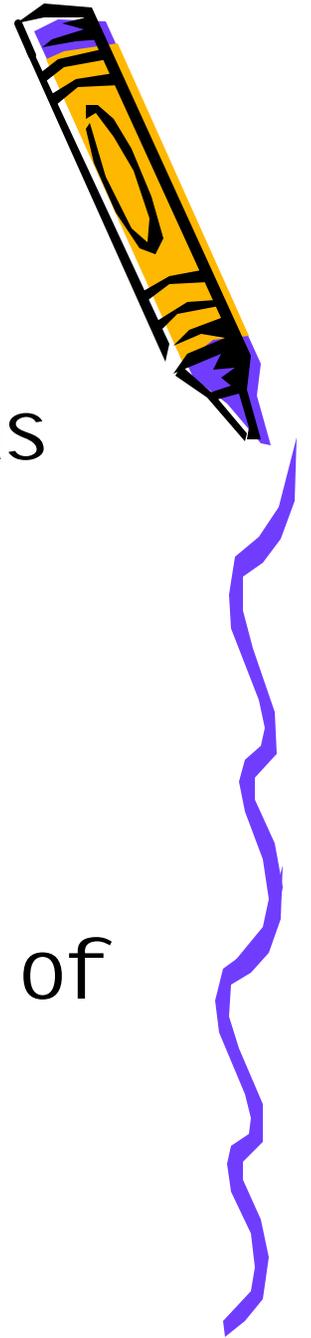
Newton's Approximation of Pi

By: Sarah Riffe
and Jen Watt



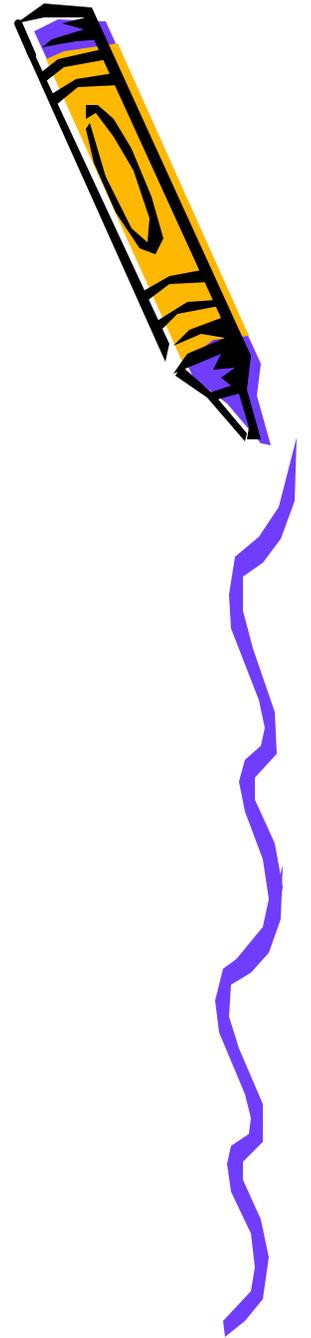
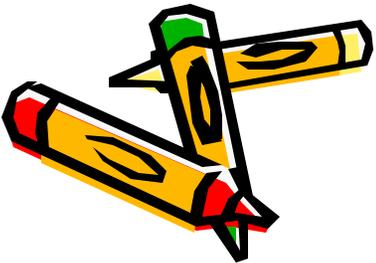
Outline

- Who was Isaac Newton? What was his life like?
- What is the history of Pi?
- What was Newton's approximation of Pi?



History of Isaac Newton

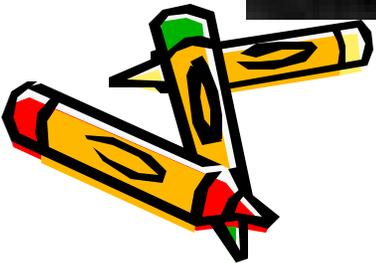
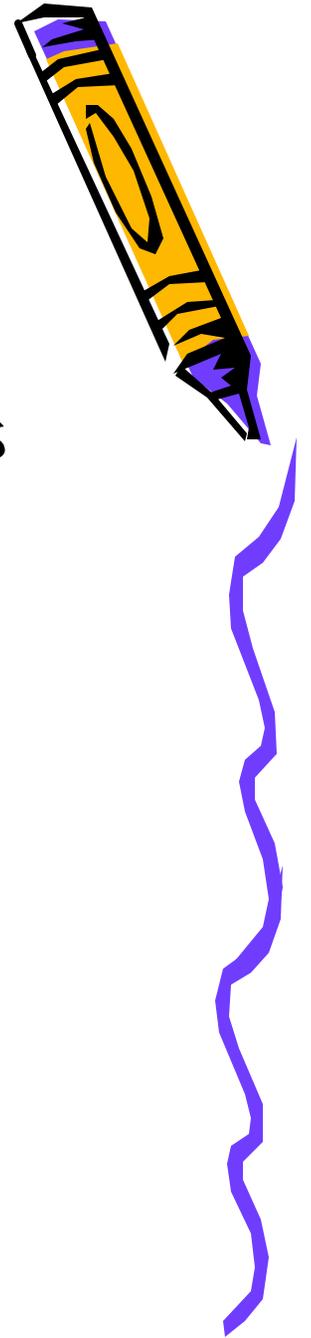
- 17th Century
 - Shift of progress in math
 - “relative freedom” of thought in Northern Europe



The Life of Newton

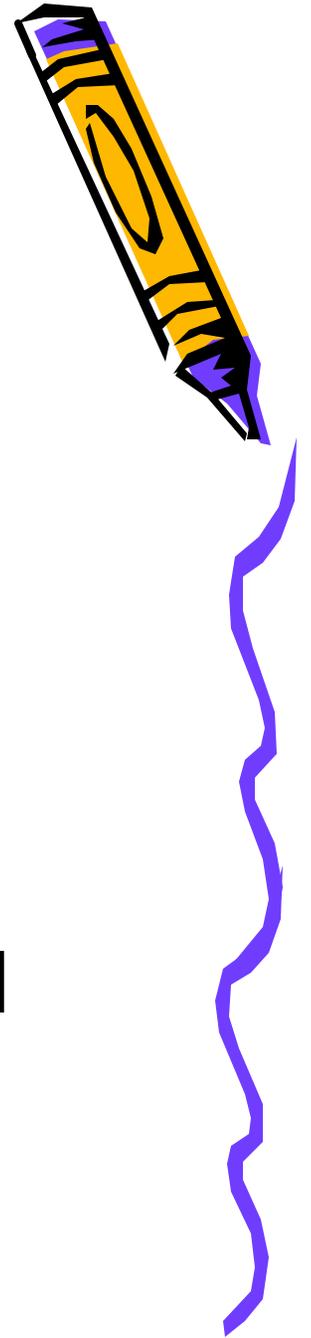
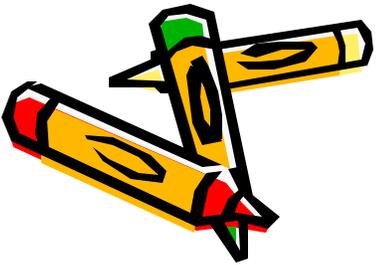


- Born: Christmas day 1642
- Died: 1727
- Raised by grandmother



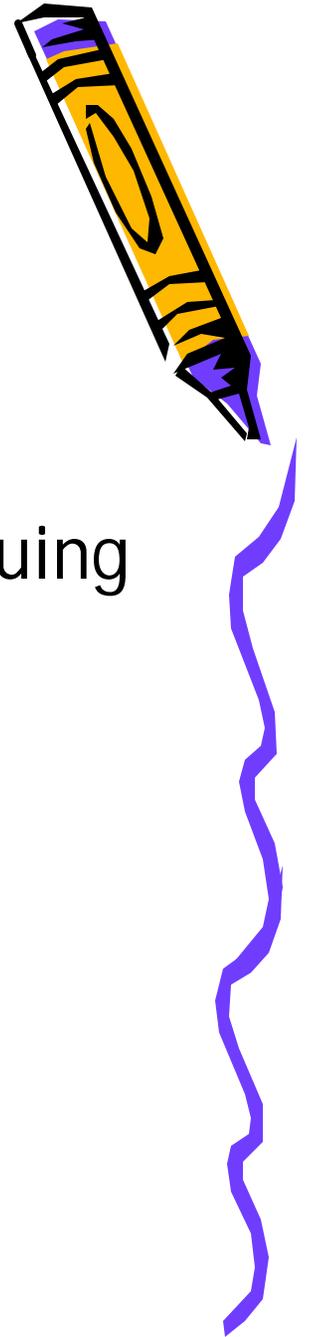
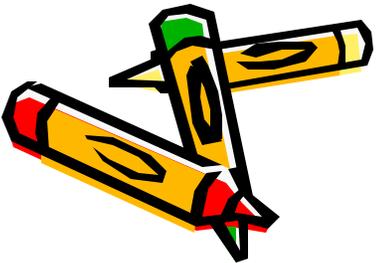
Newton's Education

- 1661
- Began at Trinity College of Cambridge University
- 1660
- Charles II became King of England
- Suspicion and hostility towards Cambridge



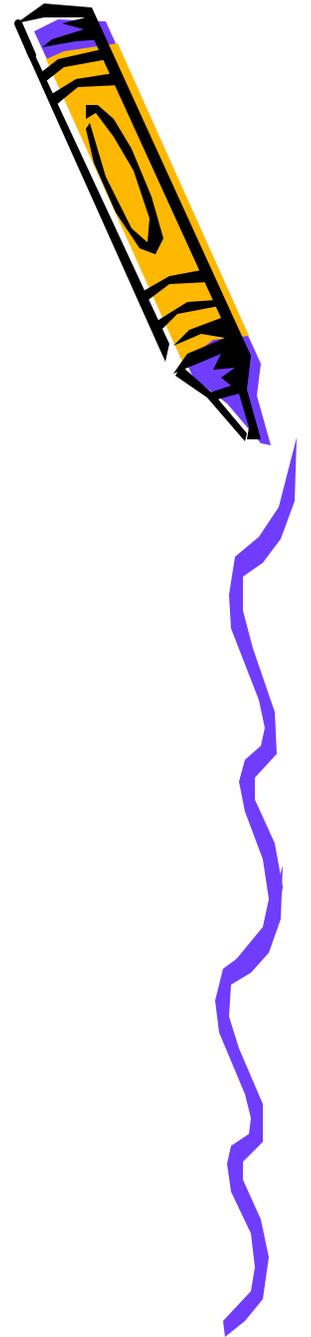
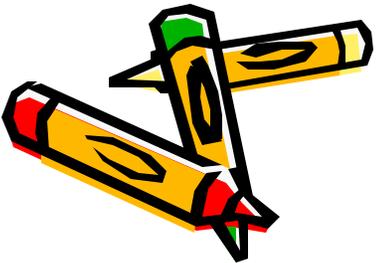
Newton, the young man

- "single minded"
 - Would not eat or sleep over an intriguing problem
- Puritan
 - Book of sins



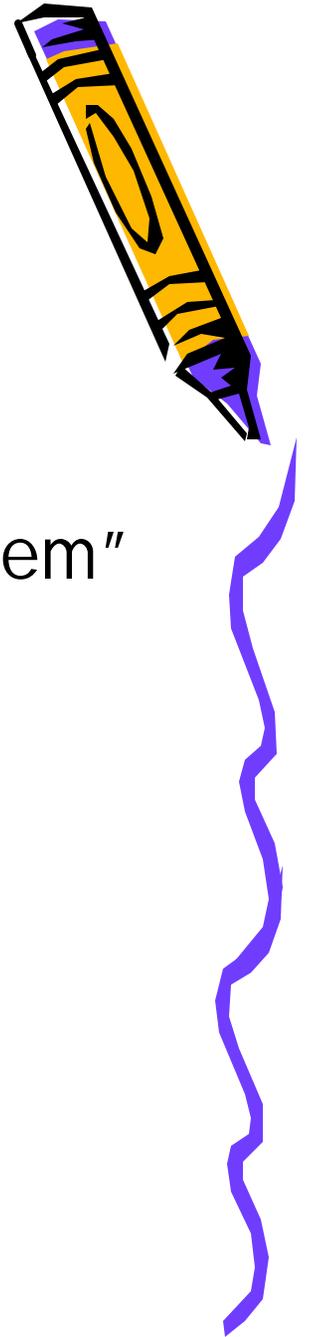
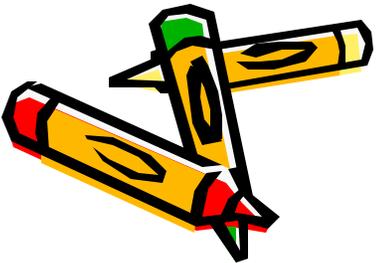
Newton's Studies

- 1664
 - Promoted to scholar at Trinity
- 1665-1666
 - Plague
 - Newton's most productive years



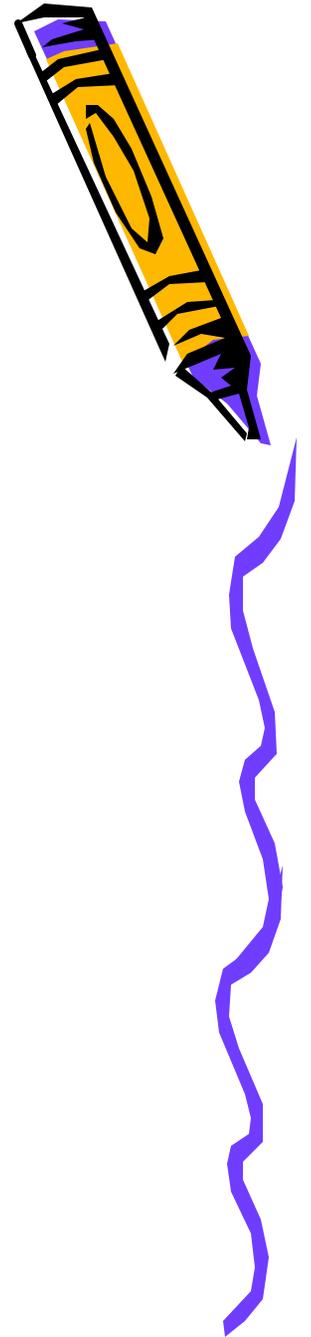
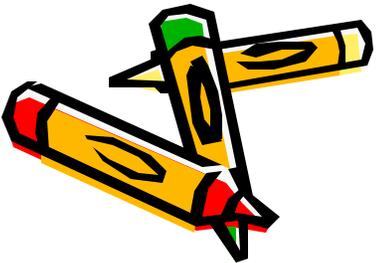
Newton's Discoveries

- 1665
 - Newton's "generalized binomial theorem"
 - led to method of fluxions
- 1666
 - Inverse method of fluxions
 - Began observations of rotation of planets



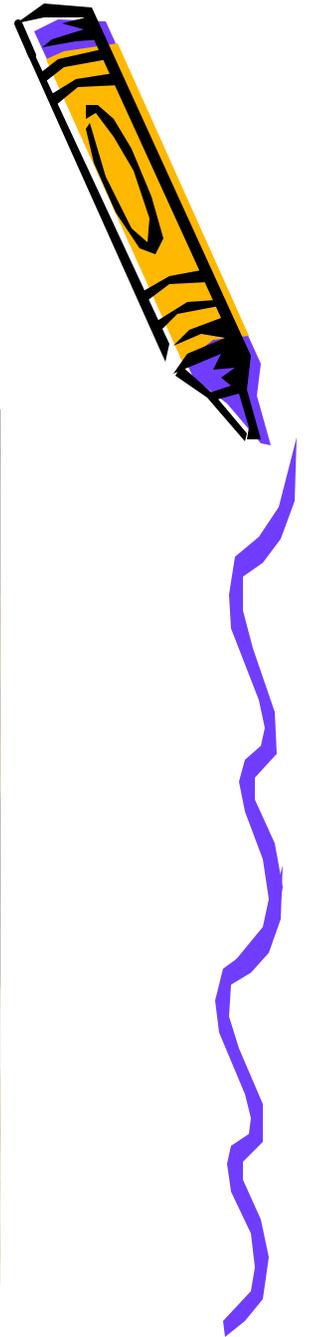
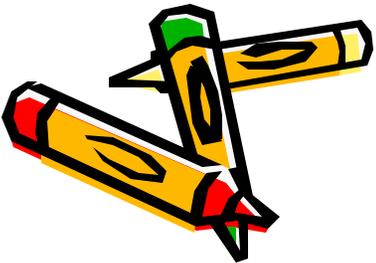
Newton's Accomplishments

- 1668
 - Finished master's degree
 - Elected fellow of Trinity College
- 1669
 - Appointed Lucasian chair of mathematics



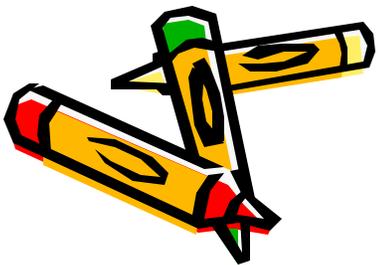
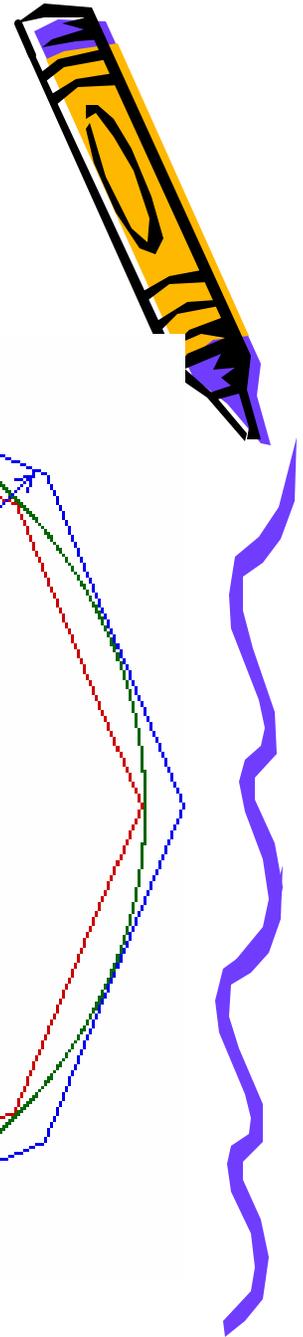
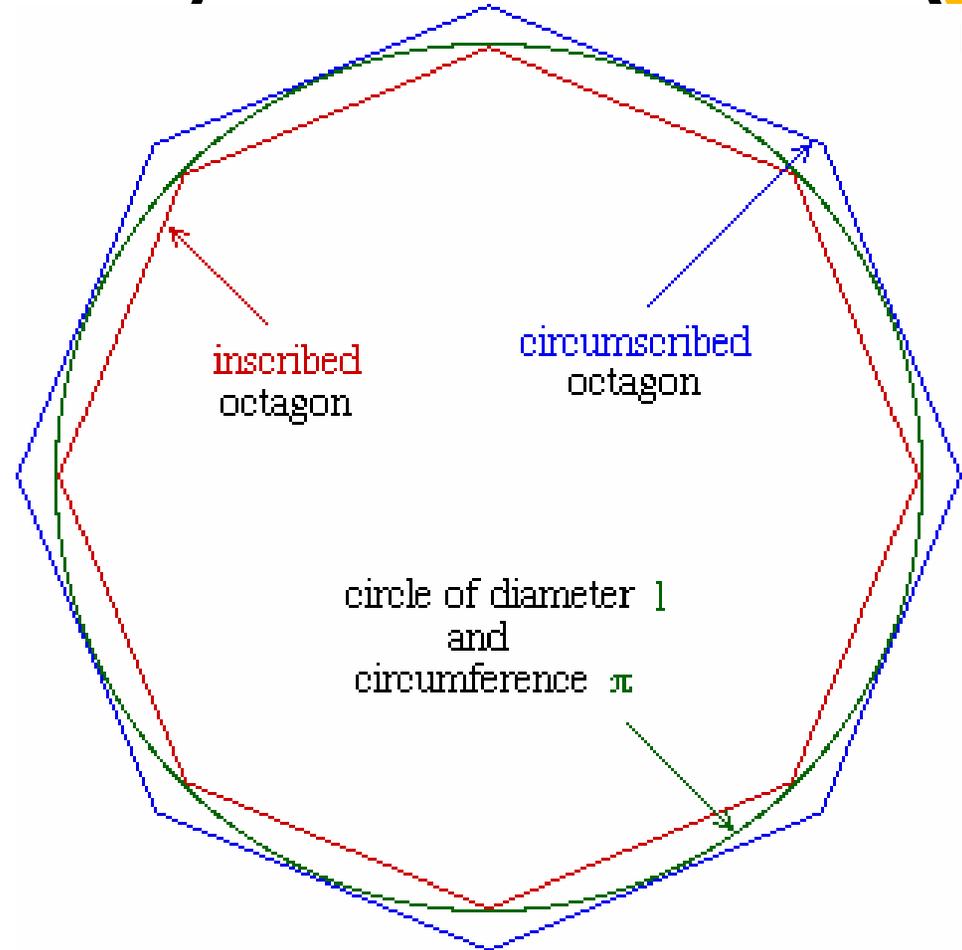
Newton's Accomplishments

- @ 1704
 - Elected President of the Royal Society
- 1705
 - Knighted by Queen Anne
- 1727
 - Buried in Westminster Abbey



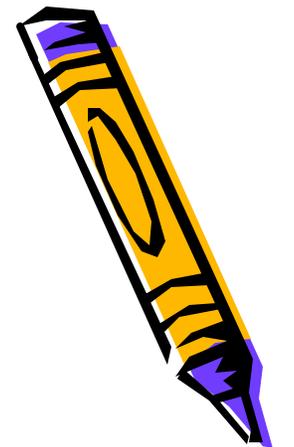
The History of Pi

- Archimedes' classical method
 - Using Polygons with inscribed And Circumscribed circles

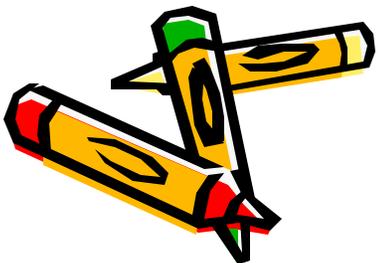


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 1415 9265 3589 7932 3846 2643 3832 7950 2884 1971 6939 9375 1058 2097 4944 5923
 0781 6406 2862 0899 8628 0348 2534 2117 0679 8214 8086 5132 8230 6647 0938 4460
 9550 5822 3175 4502 8410 2701 9385 2110 5559 6446 2294 8954
 9303 8196 4095 847 5648 2337 8678 3165 2712 0190 9145 6485
 6692 3460 4086 1326 1339 3607 2602 4914 7375 8700 6606 3155 8817
 4881 5209 2096 2354 540 5364 3678 9259 9260 0113 3093 0548 8204 6652 1384
 1469 5194 1511 0285 305 3657 5959 1953 0921 8614 9571 9326 1179 3105 1185
 4807 4462 3799 4797 0567 8575 2708 93 8183 0117 4912 9833 6733 6244
 0656 6430 8602 9463 4737 1907 0217 9860 137 07 0539 2171 7629 3176
 7523 8467 4811 6940 000 812 7145 2635 6069 785 7742 4275 7789 0091
 7363 7178 7179 0901 146 5495 8537 1050 7922 7968 9258 9235 4201
 9956 1121 2514 0 8640 3412 8136 2977 4771 3099 6051 8707 2113 4999 9998
 3729 7804 9951 0597 3173 2816 0963 1859 5024 4594 5534 6908 3026 4252 2308 2533
 4468 5035 2619 3118 8171 0100 0313 7838 7528 8658 7533 2083 8142 0617 1776 6914
 7303 5982 5349 0428 7554 6873 1159 5628 6388 2353 7875 9375 1957 7818 5778 0532
 1712 2680 6613 0019 2787 6611 1959 0921 6420 1989 3809 5257 2010 6548 5863 2788

$\pi = 3.141592653589793238462643383279502884197169399375105820974944592307816406286208998628034825342117067982148086513282306647093844609550582231754502841027019385211055596446229489549303819640958475648233786783165271201909145648566923460408613261339360726024914737587006606315588174881520920962354053643678925992600113309305488204665213841469519415110285305365759591953092186149571932611793105118548074462379947970567857527089381830117491298336733624406566430860294634737190702179860137070539217176293176752384674811694000081271452635606978577424275778900917363717871790901146549585371050792279689258923542019956112125140864034128136297747713099605187072113499999983729780499510597317328160963185950244594553469083026425223082533446850352619311881710100031378387528865875332083814206171776691473035982534904287554687311595628638823537875937519577818577805321712268066130019278766111959092164201989380952572010654858632788$

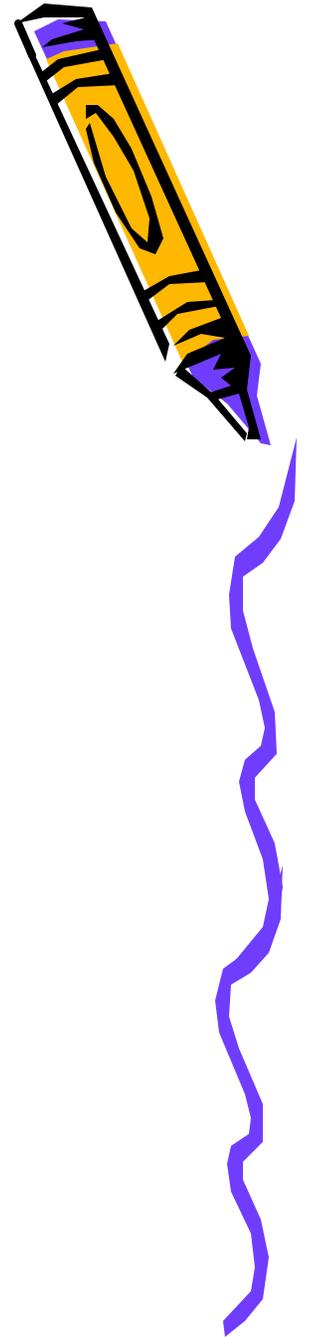
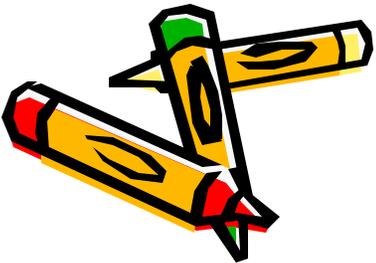


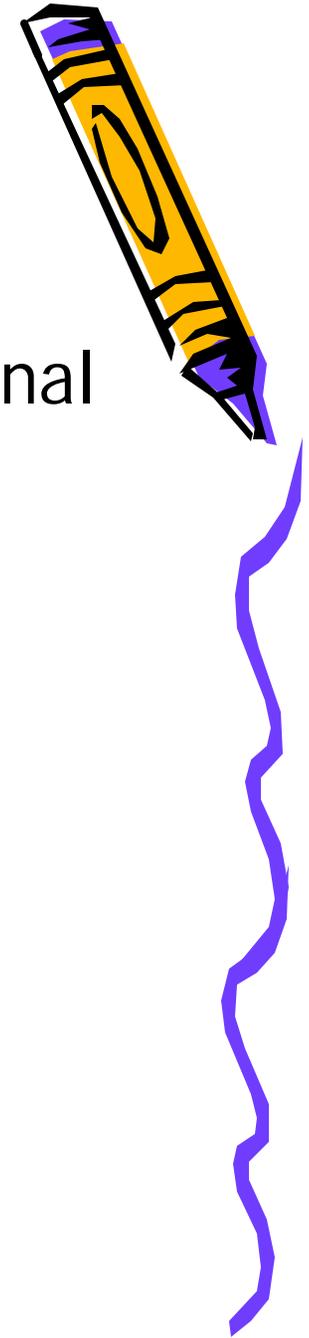
- Found Pi between 223/71 and 22/7
- =3.14



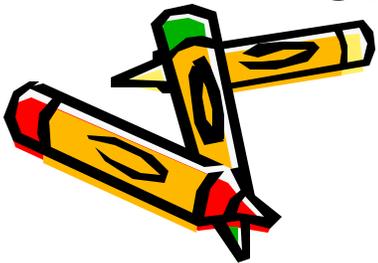
Important Dates of Pi

- 150 AD
 - First notable value for Pi by Caludius Ptolemy of Alexandria
 - $\text{Pi} = 3\ 8'30''$
 - = $377/120$
 - = 3.1416

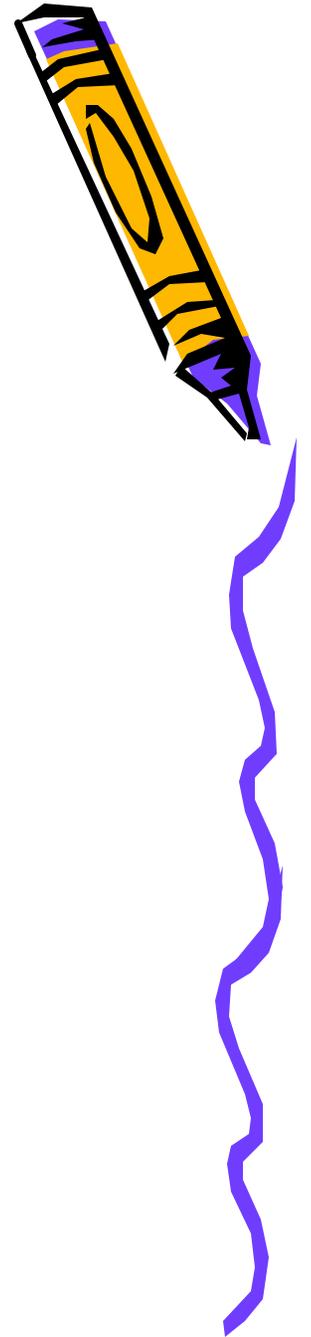
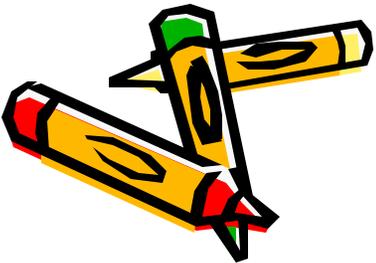




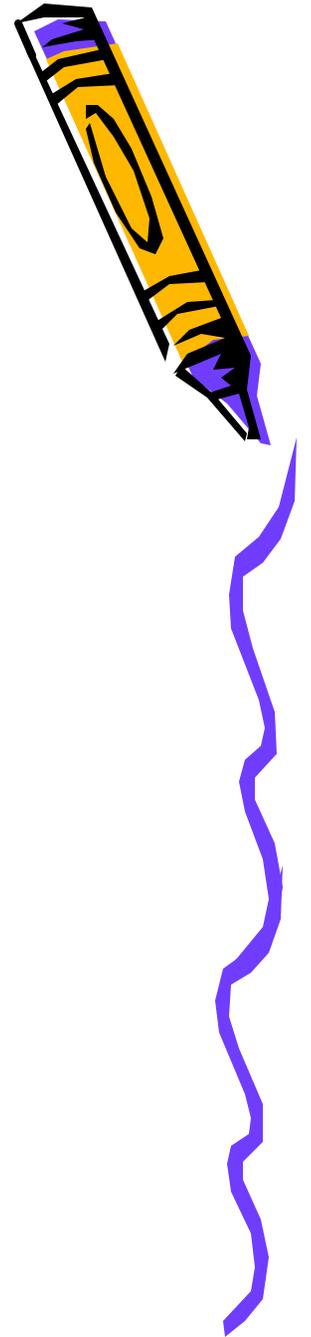
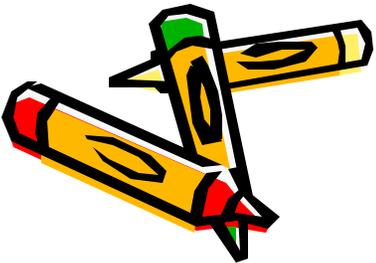
- 480 AD
 - TSU Ch'ung-chih from China gave rational approximation
 - $\text{Pi} = 355/113$
 $= 3.1415929$
- 530 AD
 - Hindu mathematician Aryabhata
 - $\text{Pi} = 62,832/20,000$
 $= 3.1416$



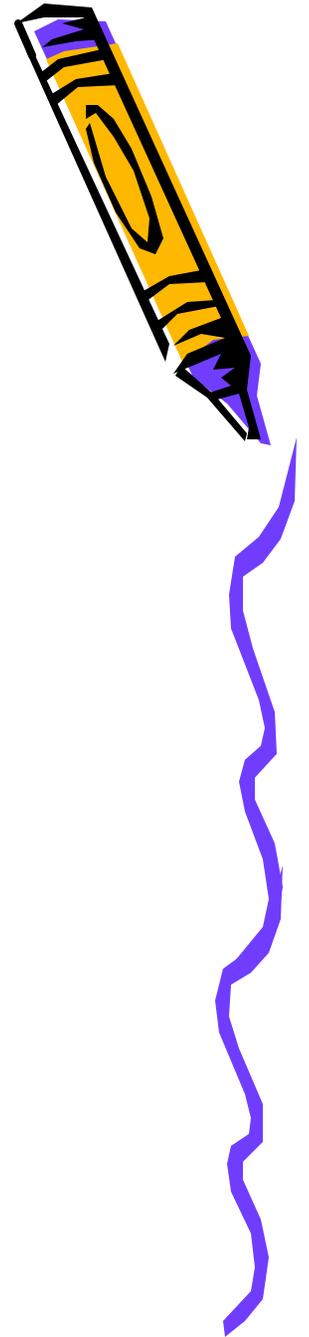
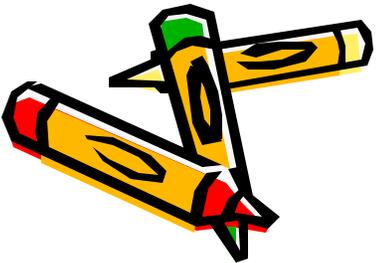
- 1150 AD
 - Bhaskara
 - $\text{Pi} = 3,927/1250$
 - $\text{Pi} = 22/7$
 - $\text{Pi} = 754/240$
 - $= 3.1416$



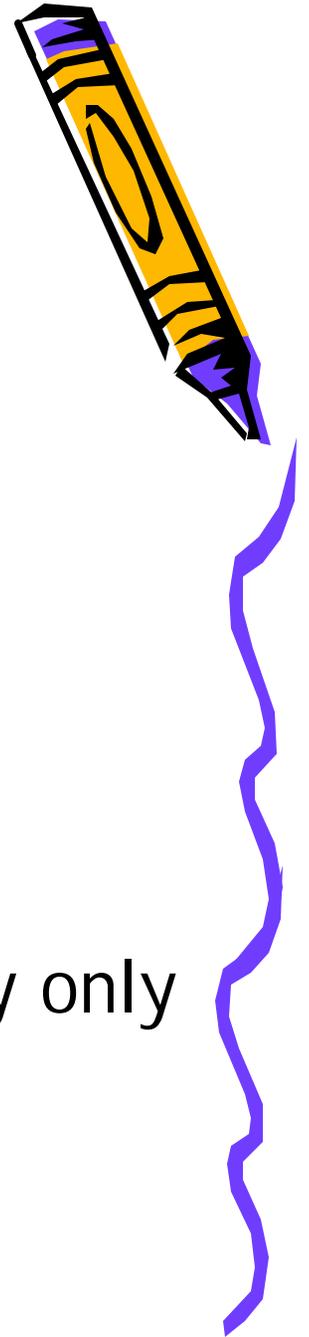
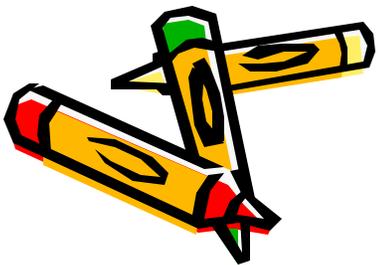
- 1429 AD
 - Al- Kashi
 - Astronomer approximated Pi to 16 decimal places
- 1579 AD
 - Francois Viete from France
 - Approximated Pi to 9 decimal places



- 1585 AD
 - Adriaen Anthoniszoon
 - Rediscovered Chinese ratio $355/113$
 - $377/120 > \pi > 333/106$
- 1593 AD
 - Adriaen Von Roomen
 - Found π to the 15th decimal place by classical method using polygons with 2^{30} sides

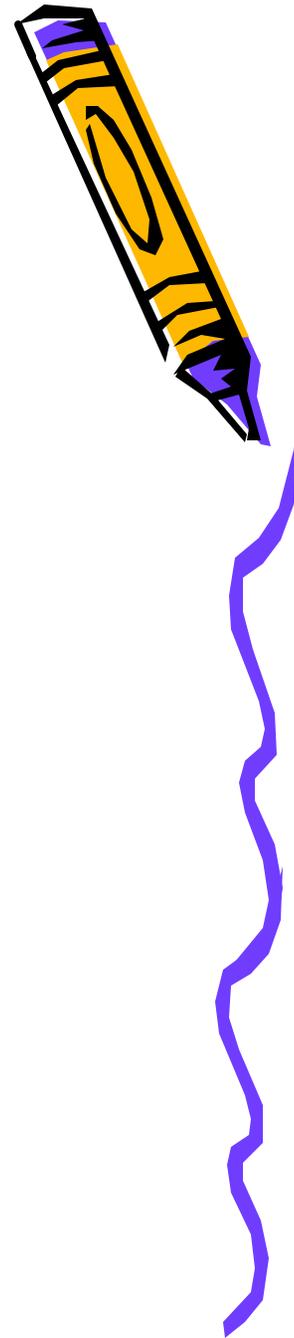
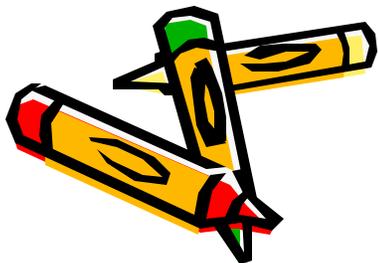


- 1610 AD
 - Ludolph Van Ceulen of the Netherlands
 - Pi ~ 30 decimal places
 - Used polygons with 2^{62} sides
- 1621 AD
 - Willebrord Snell (Dutch)
 - Able to get Ceulen's 35th decimal place by only 2^{30} side polygon

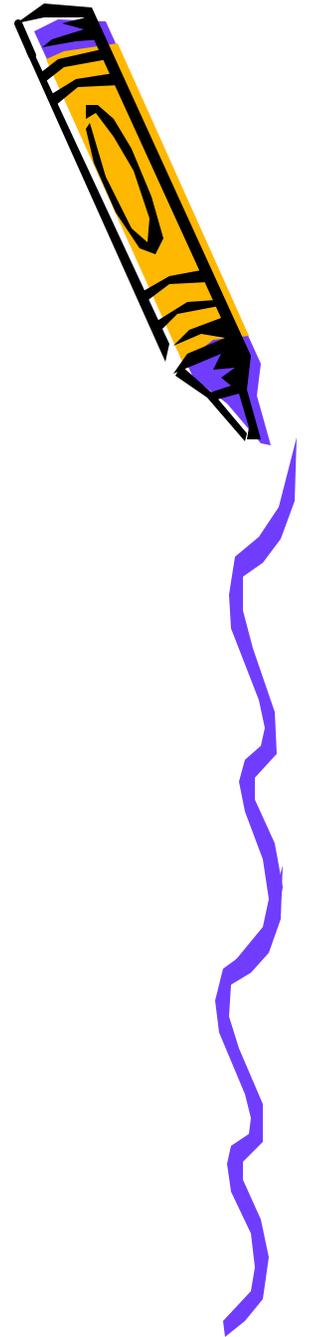
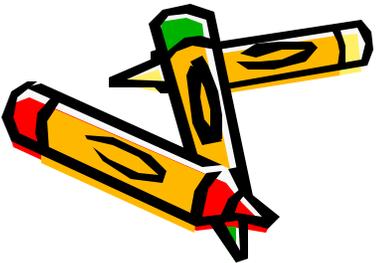


- 1630 AD
 - Grienberger
 - Pi to 39 decimal places
- 1671
 - James Gregory from Scotland obtained infinite series

$$\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \dots (-1 \leq x \leq 1)$$

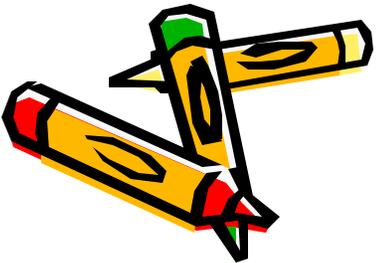
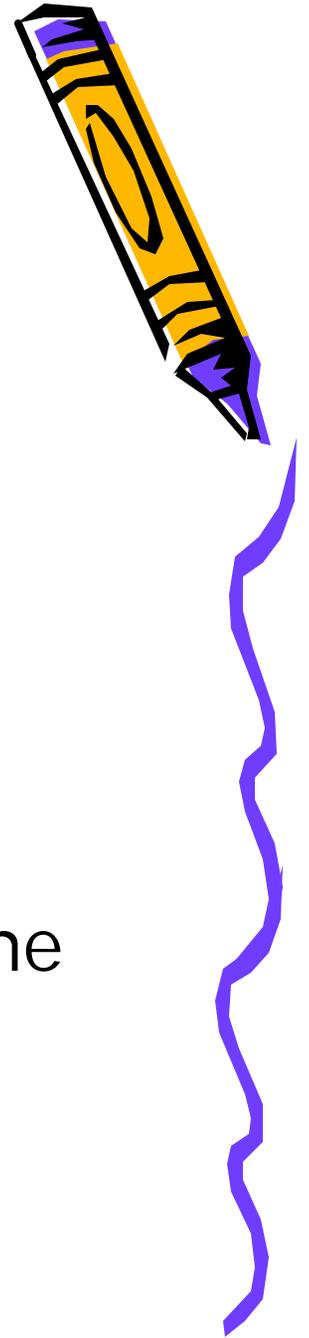


- 1699 AD
 - Abraham Sharp
 - Pi ~ 71 decimal places
- 1706 AD
 - John Machin
 - Pi ~ 100th decimal place

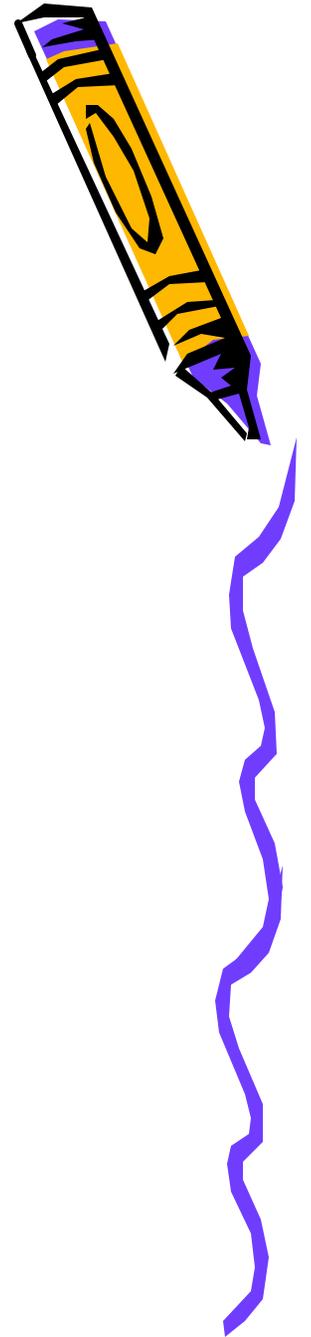
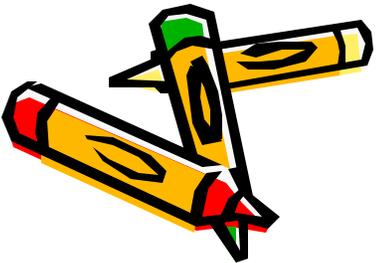


- 1719 AD
 - De Lagny of France
 - $\pi \sim 112$ decimal places

- 1737 AD
 - William Jones from England
 - First to use π symbol for ratio of the circumference to the diameter

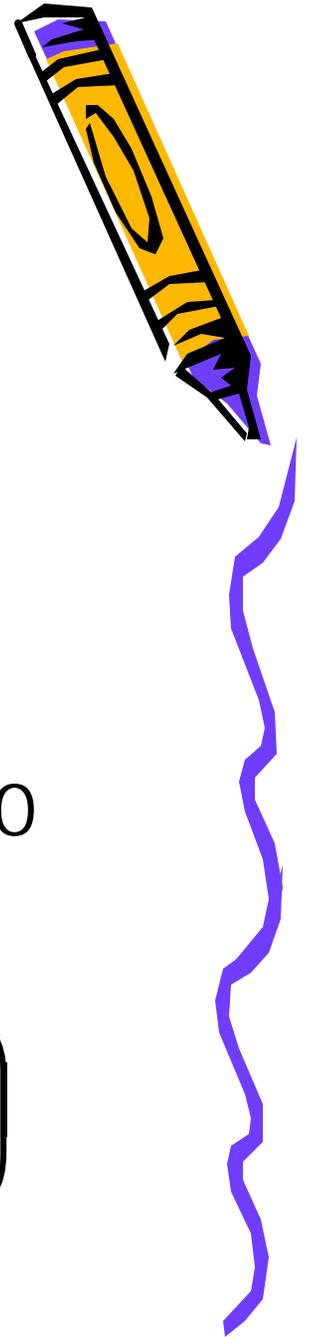
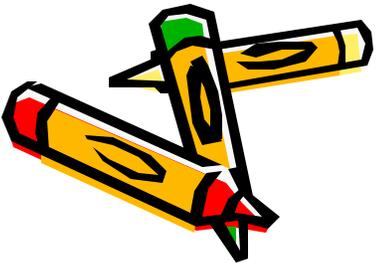


- 1767 AD
 - Johan Heinrich Lambert
 - Showed Pi is irrational
- 1794 AD
 - Adrien-Marie Legendre
 - Showed Pi-squared is irrational

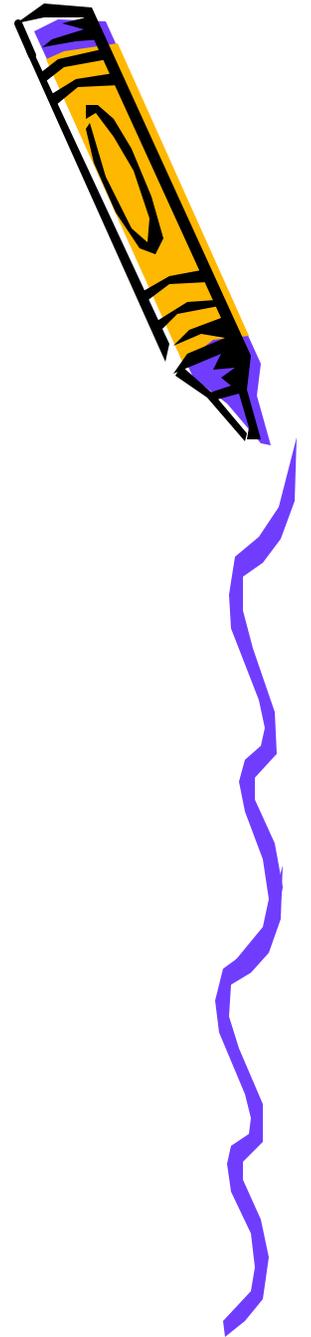
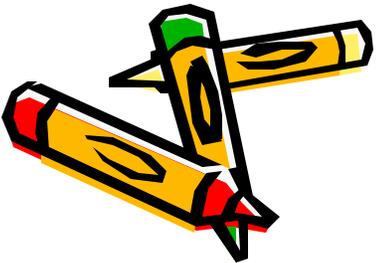


- 1841 AD
 - William Rutherford
 - Calculated Pi to 208 places
- 1844 AD
 - Zacharis Dase found Pi correct to 200 places using Gregory Series

$$\frac{p}{2} = \arctan\left(\frac{1}{2}\right) + \arctan\left(\frac{1}{5}\right) + \arctan\left(\frac{1}{8}\right)$$

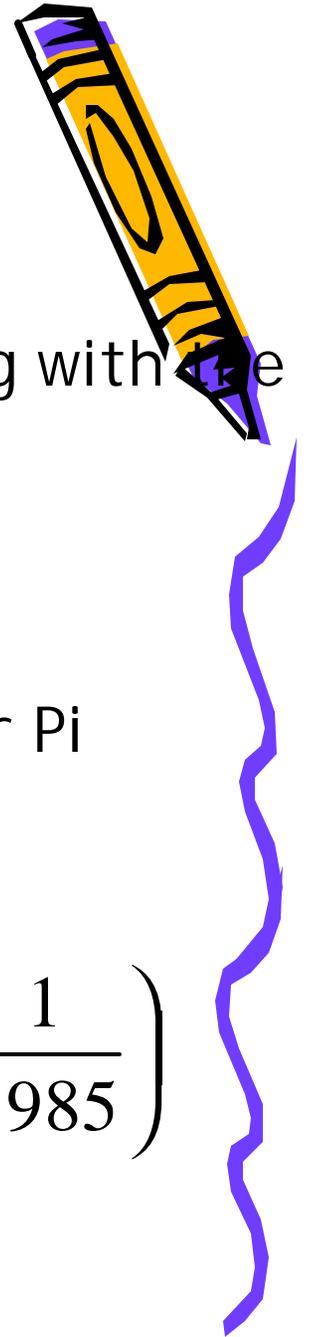
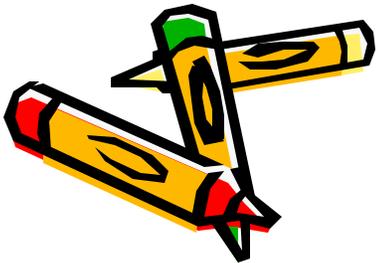


- 1853 AD
 - Rutherford returns
 - Finds Pi to 400 decimal places
- 1873 AD
 - William Shanks from England
 - Pi to 707 decimal places
- 1882 AD
 - F. Lindeman
 - Shows Pi is transcendental



- 1948
 - D.F. Ferguson of England
 - Finds errors with Shanks value of Pi starting with the 528th decimal place
 - Gives correct value to the 710th place
 - J.W. Wrench Jr.
 - Works with Ferguson to find 808th place for Pi
 - Used Machin's formula

$$\frac{\pi}{4} = 3 \arctan\left(\frac{1}{4}\right) + \arctan\left(\frac{1}{20}\right) + \arctan\left(\frac{1}{1985}\right)$$



- 1949 AD

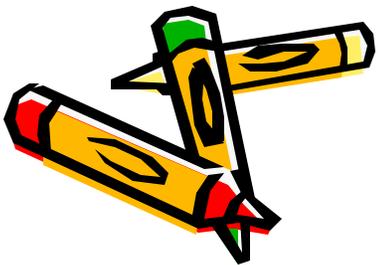
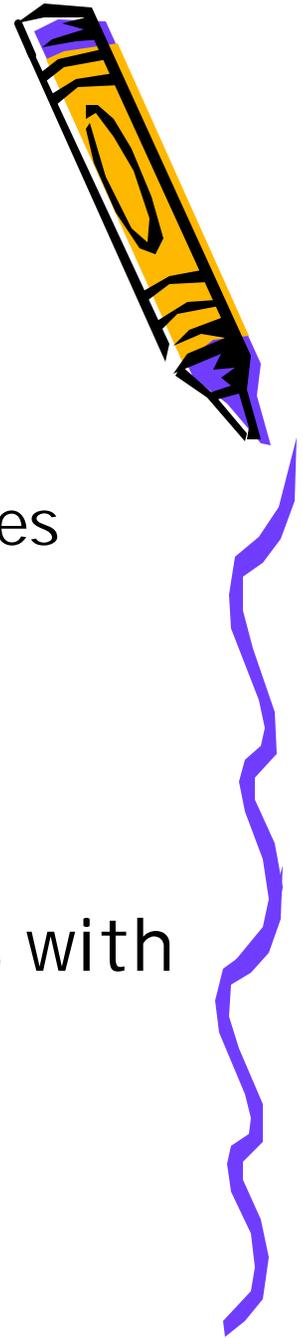
- Electronic computer - The ENIAC

- Compute Pi to the 2,037th decimal places

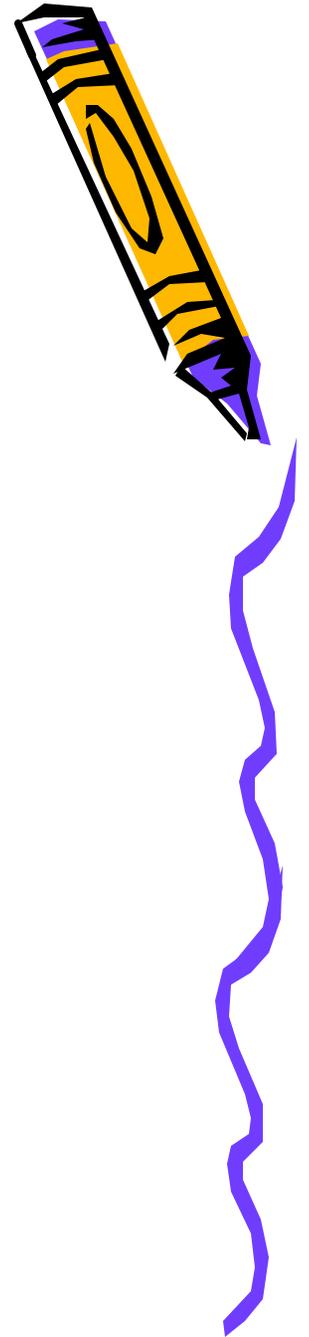
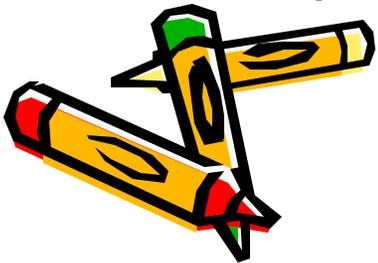
- 1959 AD

- Francois Genuys from Paris

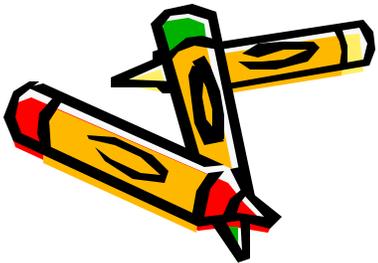
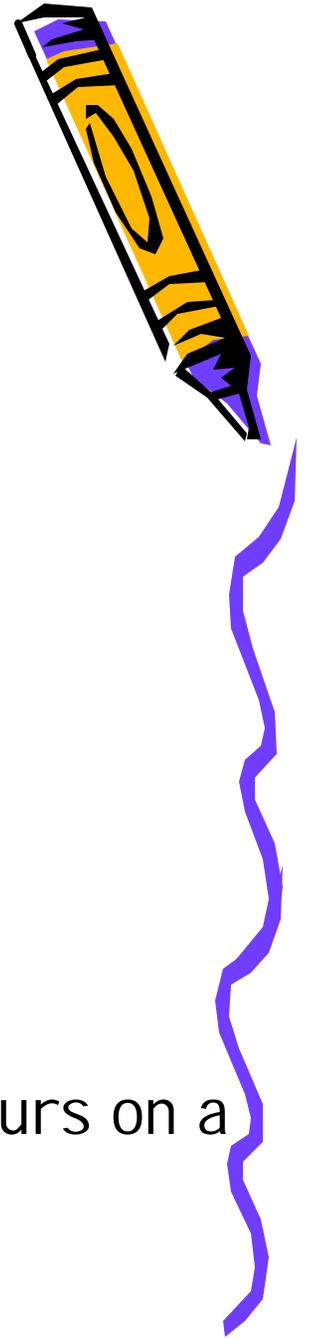
- Compute Pi to 16,167 decimal places with IBM 704

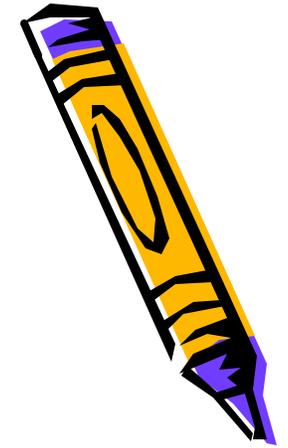


- 1961 AD
 - Wrench and Shanks of Washington D.C.
 - compute Pi to 100,265th
using IBM 7090
- 1966 AD
 - M. Jean Guilloud and co-workers
 - attained approximation for Pi
to 250,000 decimal places on a STRETCH
computer

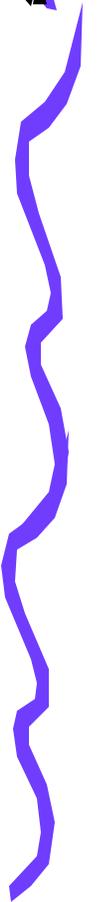
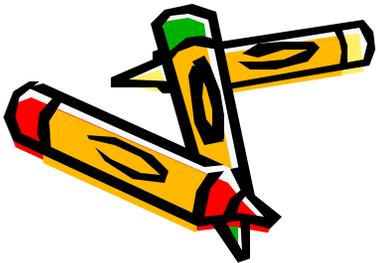


- 1967 AD
 - M. Jean Guilloud and coworkers
 - found Pi to the 500,000 places on a CDC 6600
- 1973
 - M. Jean Guilloud and coworkers found Pi to 1 millionth place on CDC 7600
- 1981 AD
 - Kazunori Miyoshi and Kazuhika Nakayma of the University of Tsukuba
 - Pi to 2 million and 38 decimal places in 137.30 hours on a FACOM M-200 computer



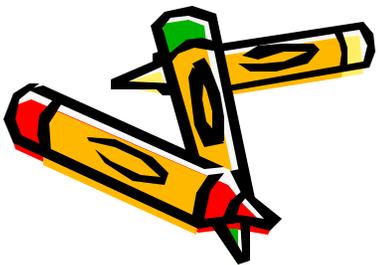
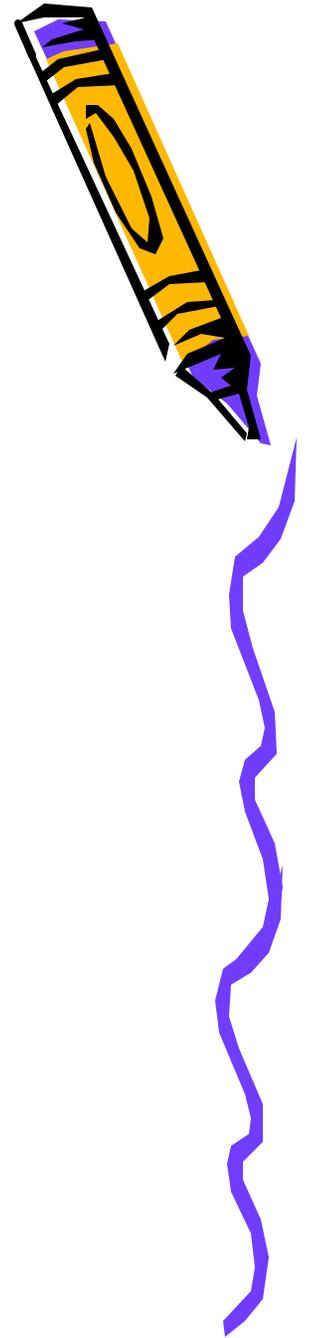


- 1986 AD
 - DH Bailey of NASA Ames Research Center ran a Cray-2 supercomputer for 28 hours
 - Got Pi to 29,360,000 decimal places
 - Yasumasa Kanada from University of Tokyo
 - Used NEC SX-2 super computer to compute Pi to 134,217,700 decimal places

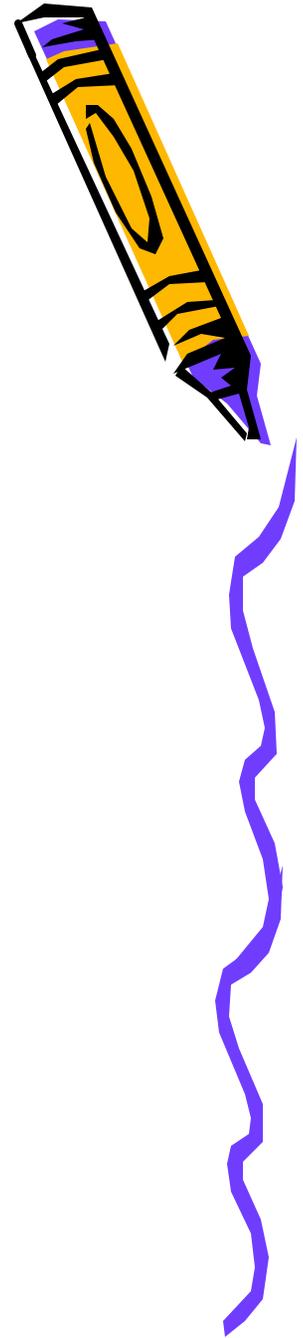


Purpose to Continue to Compute Pi

- See if digits of Pi start to repeat
 - Possible normalcy of Pi
- Valuable in computer science for designing programs



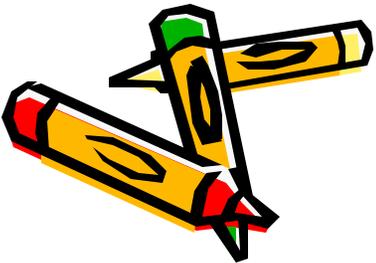
Information Already known



$$\left(x - \frac{1}{2}\right)^2 + (y - 0)^2 = \frac{1}{2}^2$$

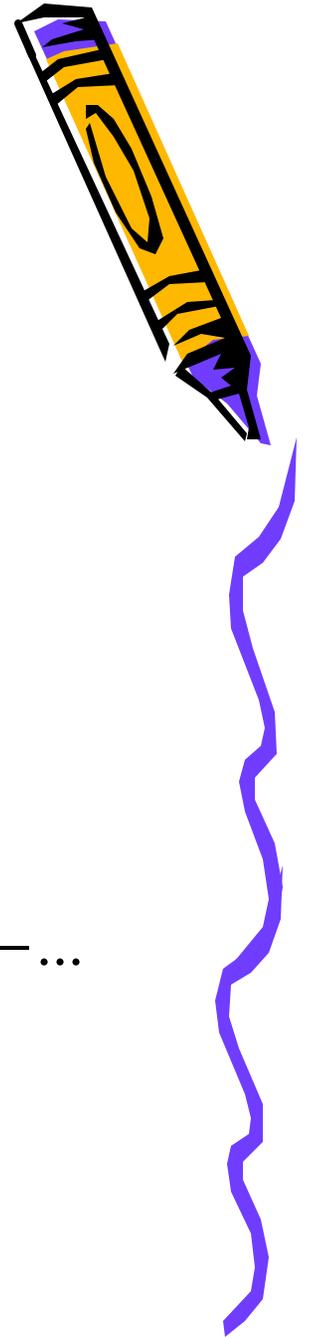
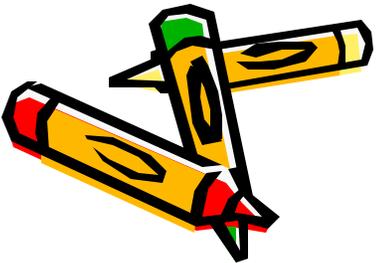
or

$$x^2 - x + \frac{1}{4} + y^2 = \frac{1}{4}$$



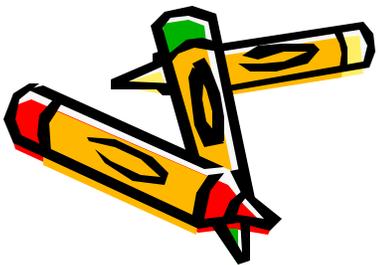
Solve for "y"

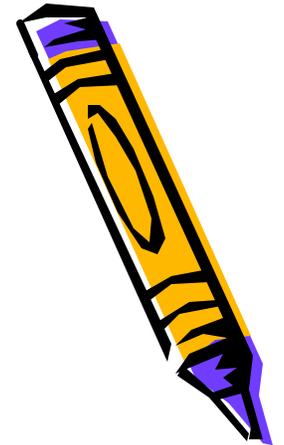
$$\begin{aligned}y &= x^{1/2} (1-x)^{1/2} \\&= x^{1/2} \left(1 - \frac{1}{2}x - \frac{1}{8}x^2 - \frac{1}{16}x^3 - \frac{5}{128}x^4 - \frac{7}{256}x^5 - \dots\right) \\&= x^{1/2} - \frac{1}{2}x^{3/2} - \frac{1}{8}x^{5/2} - \frac{1}{16}x^{7/2} - \frac{5}{128}x^{9/2} - \frac{7}{256}x^{11/2} - \dots\end{aligned}$$



Area (ABD) by fluxion

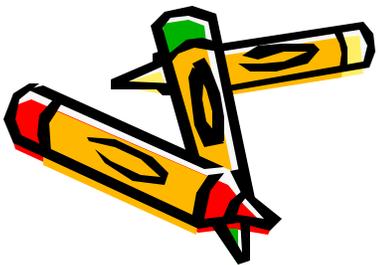
$$\begin{aligned} & \frac{2}{3}x^{3/2} - \frac{1}{2}\left(\frac{2}{5}x^{5/2}\right) - \frac{1}{8}\left(\frac{2}{7}x^{7/2}\right) - \frac{1}{16}\left(\frac{2}{9}x^{9/2}\right) - \dots \\ & = \frac{2}{3}x^{3/2} - \frac{1}{5}x^{5/2} - \frac{1}{28}x^{7/2} - \frac{1}{72}x^{9/2} - \frac{5}{704}x^{11/2} - \dots \end{aligned}$$





$$\left(\frac{1}{4}\right)^{3/2} = \left(\sqrt{\frac{1}{4}}\right)^3 = \frac{1}{8} \text{---}, \text{---}\left(\frac{1}{4}\right)^{5/2} = \left(\sqrt{\frac{1}{4}}\right)^5 = \frac{1}{32} \text{---}\dots$$

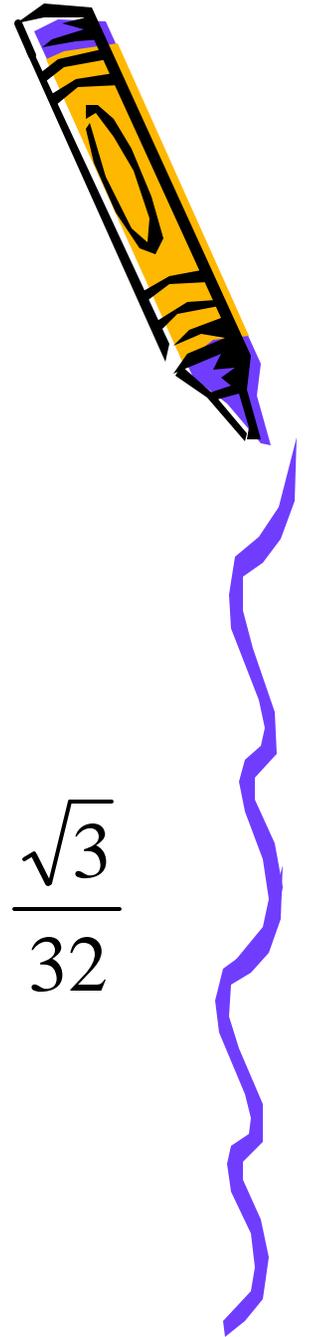
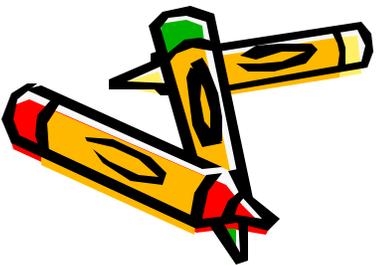
$$\frac{1}{12} - \frac{1}{160} - \frac{1}{3584} - \frac{5}{1441792} \dots - \frac{429}{163208757248} = .07677310678$$



Area (ABD) by geometry

$$\overline{BD} = \sqrt{\left(\frac{1}{2}\right)^2 - \left(\frac{1}{4}\right)^2} = \sqrt{\frac{3}{16}} = \frac{\sqrt{3}}{4}$$

$$\text{Area}(\triangle DBC) = \frac{1}{2}(\overline{BC})x(\overline{BD}) = \frac{1}{2}\left(\frac{1}{4}\right)\left(\frac{\sqrt{3}}{4}\right) = \frac{\sqrt{3}}{32}$$

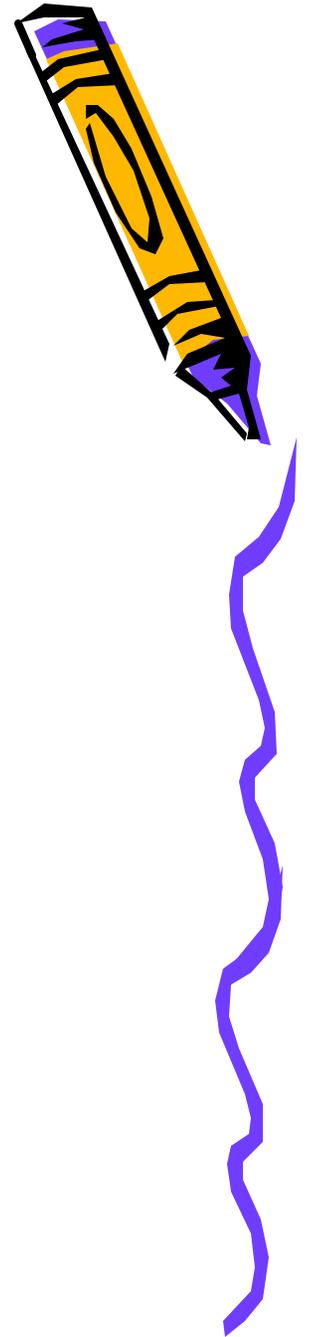
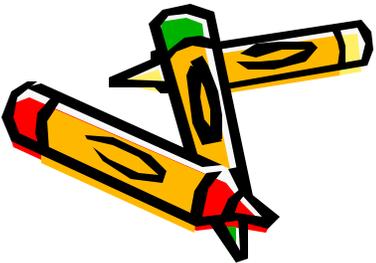


$$\text{Area}(\text{sector}) = \frac{1}{3} \text{Area}(\text{semicircle})$$

$$= \frac{1}{3} \left(\frac{1}{2} \cdot \mathbf{p} \cdot r^2 \right)$$

$$= \frac{1}{3} \left[\frac{1}{2} \mathbf{p} \left(\frac{1}{2} \right)^2 \right]$$

$$= \frac{\mathbf{p}}{24}$$





$$\text{Area}(ABD) = \text{Area}(\text{sector}) - \text{Area}(\Delta DBC)$$

$$= \frac{p}{24} - \frac{\sqrt{3}}{32}$$

$$p \approx 24 \left(.07677310678 + \frac{\sqrt{3}}{32} \right) = 3.141592668\dots$$

