MA 361 - 02/27/2012 FIRST MIDTERM	Spring 2012 A. Corso	Name: ______________________ ______________________

PLEASE, BE NEAT AND SHOW ALL YOUR WORK; JUSTIFY YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
Bonus	5	
TOTAL	55	/ 50

1. (a) Write the arithmetic expression

$$\frac{2-4i}{i(3+i)}$$

in the form $a+ib$ for $a, b \in \mathbb{R}$.

(b) Find all solutions in $\mathbb{C}$ of the equation $z^3 = -27i$.

pts: / 10

2. Choose one of the following problems:

(a) Determine whether the map

$$\varphi\colon\ (M_2(\mathbb{R}), \cdot) \longrightarrow (\mathbb{R}, \cdot),$$

where $\varphi(A) = \det(A)$, is an isomorphism of binary structures.
Explain.

(b) Let F be the set of all functions f mapping $\mathbb{R}$ into $\mathbb{R}$ that have derivatives of all orders. Determine whether the map

$$\varphi\colon\ (F, +) \longrightarrow (F, +),$$

where $\varphi(f)(x) = \dfrac{d}{dx}\displaystyle\int_0^x f(t)\, dt$, is an isomorphism of binary structures.
Explain.

pts: / 10

3. The map $\varphi : \mathbb{Q} \longrightarrow \mathbb{Q}$ defined by $\varphi(x) = 3x - 1$ for $x \in \mathbb{Q}$ is one-to-one and onto $\mathbb{Q}$. Give the definition of a binary operation $*$ on $\mathbb{Q}$ such that φ is an isomorphism mapping

(a) $(\mathbb{Q}, +)$ onto $(\mathbb{Q}, *)$,

(b) $(\mathbb{Q}, *)$ onto $(\mathbb{Q}, +)$.

pts: / 10

4. (a) Determine whether the binary operation $*$ defined by

$$a * b = \frac{ab}{3}$$

gives a group structure on the set $\mathbb{Q}^*$ of nonzero rational numbers.

(b) Determine whether the binary operation $*$ defined by

$$a * b = \frac{a}{b}$$

gives a group structure on the set $\mathbb{R}^*$ of nonzero real numbers.

pts: / 10

5. Let $(G, *)$ be a group.

(a) Show that $(g_1 * g_2)^2 = g_1^2 * g_2^2$, for some $g_1, g_2 \in G$, if and only if $g_1 * g_2 = g_2 * g_1$.

(b) Conclude that the map $f\colon\ G \longrightarrow G$, defined by setting $f(g) = g^2$ for all $g \in G$, is an homomorphism if and only if G is abelian.

pts: / 10

Bonus. Choose one of the following problems:

(a) Let $f\colon A \longrightarrow B$ be a surjective map of sets. Prove that the relation

$$a \sim b \qquad \text{if and only if} \qquad f(a) = f(b)$$

is an equivalence relation. Describe the equivalence classes of $\sim$?

(b)

$*$	a	b	c	d
a	a	b	c	d
b	b	a	c	d
c	c	d	c	d
d				

The above table can be completed to define an associative binary operation $*$ on $S = \{a, b, c, d\}$. Assume this is possible and compute the missing entries.

pts: /5