MA 361 - 02/27/2012 Spring 2012 FIRST MIDTERM A. Corso	Name:
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PLEASE, BE NEAT AND SHOW ALL YOUR WORK; JUSTIFY YOUR ANSWER.

Problem Number	Possible Points	Points Earned
1	10	
2	10	
3	10	
4	10	
5	10	
Bonus	5	
TOTAL	55	/50

**1.** (a) Write the arithmetic expression

$$\frac{2-4i}{i(3+i)}$$

in the form a + ib for  $a, b \in \mathbb{R}$ .

(b) Find all solutions in  $\mathbb{C}$  of the equation  $z^3 = -27i$ .

- **2.** Choose one of the following problems:
  - (a) Determine whether the map

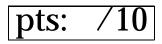
$$\varphi \colon (M_2(\mathbb{R}), \cdot) \longrightarrow (\mathbb{R}, \cdot),$$

where  $\varphi(A) = \det(A)$ , is an isomorphism of binary structures. Explain.

(*b*) Let *F* be the set of all functions *f* mapping  $\mathbb{R}$  into  $\mathbb{R}$  that have derivatives of all orders. Determine whether the map

$$\varphi \colon (F,+) \longrightarrow (F,+),$$

where  $\varphi(f)(x) = \frac{d}{dx} \int_0^x f(t) dt$ , is an isomorphism of binary structures. Explain.



- **3.** The map  $\varphi : \mathbb{Q} \longrightarrow \mathbb{Q}$  defined by  $\varphi(x) = 3x 1$  for  $x \in \mathbb{Q}$  is one-to-one and onto  $\mathbb{Q}$ . Give the definition of a binary operation \* on  $\mathbb{Q}$  such that  $\varphi$  is an isomorphism mapping
  - (a)  $(\mathbb{Q}, +)$  onto  $(\mathbb{Q}, *)$ ,
  - (b)  $(\mathbb{Q}, *)$  onto  $(\mathbb{Q}, +)$ .



**4.** (a) Determine whether the binary operation \* defined by

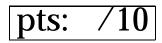
$$a * b = \frac{ab}{3}$$

gives a group structure on the set  $\mathbb{Q}^\ast$  of nonzero rational numbers.

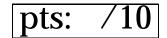
 $(b)\,$  Determine whether the binary operation \* defined by

$$a * b = \frac{a}{b}$$

gives a group structure on the set  $\mathbb{R}^*$  of nonzero real numbers.



- **5.** Let (G, \*) be a group.
  - (a) Show that  $(g_1 * g_2)^2 = g_1^2 * g_2^2$ , for some  $g_1, g_2 \in G$ , if and only if  $g_1 * g_2 = g_2 * g_1$ .
  - (b) Conclude that the map  $f: G \longrightarrow G$ , defined by setting  $f(g) = g^2$  for all  $g \in G$ , is an homomorphism if and only if G is abelian.



Bonus. Choose one of the following problems:

(a) Let  $f: A \longrightarrow B$  be a surjective map of sets. Prove that the relation

$$a \sim b$$
 if and only if  $f(a) = f(b)$ 

is an equivalence relation. Describe the equivalence classes of  $\sim$ ?

(b)

*	a	b	c	d
a	a	b	c	d
b	b	a	c	d
С	c	d	c	d
d				

The above table can be completed to define an associative binary operation \* on  $S = \{a, b, c, d\}$ . Assume this is possible and compute the missing entries.

