NAME: _____

TEST 1 **Math 511 (Cowen)** **28 February 1997**

There are 7 pages, 7 questions, and 100 points on this test.
NO CALCULATORS

(10 points) 1. Find all solutions of the following system:

$$\begin{cases} w + 2x - y + 2z = 4 \\ w + x + 2y - z = -2 \\ w + 3x - 4y + 5z = 10 \end{cases}$$

If there is a unique solution or no solution, say so. If there are infinitely many solutions, find the general solution and *find three solutions* explicitly.

(15 points) 2. Consider the following two systems of equations.

$$(H)\begin{cases} a+ & b+2c- & d+ & e- & f=0 \\ 2a+ & b- & c+ & d-2e & =0 \\ 2a+ & b- & c+ & d & -2f=0 \\ a+ & 3b-2c+3d- & e- & f & =0 \\ 2a+ & b & +\ d-2e- & f & =0 \end{cases}$$

and

$$(N)\begin{cases} a+ & b+2c- & d+ & e- & f=3 \\ 2a+ & b- & c+ & d-2e & =6 \\ 2a+ & b- & c+ & d & -2f=2 \\ a+ & 3b-2c+3d- & e- & f & =3 \\ 2a+ & b & +\ d-2e- & f & =6 \end{cases}$$

The vector $(1,-1,1,2,1,1)$ is a solution of system (H).
The vector $(2,1,1,0,-1,1)$ is a solution of system (N).
Use theorems you know (i.e. do *not* try to solve the systems completely!) to answer the following questions.

(a) Find two other non-trivial solutions of (H).

(b) Find two other solutions of (N).

(15 points) 3. Let $v_1 = (1, -1, 2)$, $v_2 = (-1, 2, -1)$, $v_3 = (2, -1, 5)$, and $v_4 = (1, 1, 5)$.

(a) Explain how you can tell that $\{v_1, v_2, v_3, v_4\}$ is a linearly dependent set without doing any calculations.

(b) Write one of the vectors as a linear combination of the rest.

4. Let

$$A = \begin{pmatrix} 1 & 2 & -1 & 0 \\ 2 & 1 & 1 & 3 \\ 1 & -1 & 2 & 3 \end{pmatrix}$$

(15 points)

(a) Find a basis for the nullspace of A.

(b) Find a basis for the range of A.

(c) What is the dimension of the range of A', $\mathcal{R}(A')$?

(d) What is the dimension of the nullspace of A', $\mathcal{N}(A')$?

(15 points) 5. The matrix C is

$$C = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 2 & -1 \\ 1 & -4 & 3 \end{pmatrix}$$

and the matrix D is a 3×5 matrix with rows D_1, D_2, and D_3.

(a) Write D as a block matrix, blocked in rows, and find CD as a block matrix, blocked in rows.

(b) Suppose, now, that the rank of D is 2 and $D_2 = -2D_1$. Find a basis for the row space of D.

(c) Suppose, as in part (b), that the rank of D is 2 and $D_2 = -2D_1$. Find a basis for the row space of CD.

(15 points) 6. Let $\mathcal{E} = \{e_1, e_2\}$ be the standard basis for $\mathbf{R}^2$ and let $\mathcal{B} = \{u_1, u_2\}$ be basis $u_1 = (3, 1)$ and $u_2 = (5, 2)$.

(a) Find the matrix for changing coordinates from the basis $\mathcal{B}$ into coordinates in the basis $\mathcal{E}$.

(b) Find the matrix for changing coordinates from the basis $\mathcal{E}$ into coordinates in the basis $\mathcal{B}$.

(c) Let $v = 2u_1 + 3u_2$. What are the coordinates of v with respect to the basis $\mathcal{B}$?

(d) Let $v = 2u_1 + 3u_2$ as above. What are the coordinates of v with respect to the basis $\mathcal{E}$?

(e) Let $w = (3, 4)$. What are the coordinates of w with respect to the basis $\mathcal{B}$?

(15 points) 7. (a) Prove the following theorem:

Let A be an $m \times n$ matrix. If B is an $n \times m$ matrix so that $BA = I$, then $\mathcal{N}(A) = (0)$ (that is, the null space of A is the zero subspace).

(b) Prove the following theorem:
(You may use the result of part (a) even if you did not answer part (a)!)

Let A be an $m \times n$ matrix.
If B is an $n \times m$ matrix so that $BA = I$, then the columns of A are linearly independent.