Homework Assignment # 1 Due Date: July 3
Chapter 1: Matrices and Gaussian Elimination

Linear Algebra and Applications

(Mathematics 642:550:01 – Summer 1995)

Exercise #1: Solve the following system of equations by Gauss elimination:

$$\left\{\begin{array}{rcrcrcrcr} -2x_1 & + & 2x_2 & - & 4x_3 & - & 6x_4 & = & -4 \\ -3x_1 & + & 6x_2 & + & 3x_3 & - & 15x_4 & = & -3 \\ 5x_1 & - & 8x_2 & - & x_3 & + & 17x_4 & = & 9 \\ x_1 & + & x_2 & + & 11x_3 & + & 7x_4 & = & 7 \end{array}\right. .$$

Exercise #2: Consider the following system of three equations in three unknowns:

$$\left\{\begin{array}{rcrcrcl} u & - & 2v & + & 3w & = & 1 \\ 2u & + & kv & + & 6w & = & 6 \\ -u & + & 3v & + & (k-3)w & = & 0 \end{array}\right. .$$

Show that:

(a) If $k = 0$, then the system has an infinite number of solutions.

(b) For another specific value of k, which you must find, the system has no solutions.

(c) For all other values of k, the system has a unique solution.

Exercise #3: Consider the matrices

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 8 & 9 & 19 \\ 2 & 2 & 5 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Factor the matrix A into $A = LU$. Solve $Ax = b$ by solving the triangular systems $Lc = b$ and $Ux = c$. What part of A^{-1} have you found with this particular b?

Exercise #4: Find the symmetric factorization $A = LDL^{\mathrm{T}}$ of

$$A = \begin{bmatrix} a & b \\ b & c \end{bmatrix}.$$

Exercise #5: Find the value for c in the following n by n inverse:

$$\text{if} \quad A = \begin{bmatrix} n & -1 & \cdots & -1 \\ -1 & n & \cdots & -1 \\ \vdots & \vdots & & \vdots \\ -1 & -1 & \cdots & n \end{bmatrix} \quad \text{then} \quad A^{-1} = \frac{1}{n+1} \begin{bmatrix} c & 1 & \cdots & 1 \\ 1 & c & \cdots & 1 \\ \vdots & \vdots & & \vdots \\ 1 & 1 & \cdots & c \end{bmatrix}.$$

Exercise #6: If A and B are n by n invertible matrices, show that $I_n - AB$ is invertible if and only if $I_n - BA$ is invertible. Hint: $B(I_n - AB) = (I_n - BA)B$.