Homework Assignment # 4 Due Date: July 27
Chapter 4: Determinants

Linear Algebra and Applications

(Mathematics 642:550:01 – Summer 1995)

Exercise #1: If every row of an $n \times n$ matrix A adds up to zero, what can you say about det A? Why?

Exercise #2: Show that

$$\det \begin{bmatrix} x & y & 1 \\ x_1 & x_2 & 1 \\ y_1 & y_2 & 1 \end{bmatrix} = 0$$

represents the equation of the line in the xy-plane through the distinct points (x_1, y_1) and (x_2, y_2).

Exercise #3: Compute the cofactor matrix A_{cof} and the inverse A^{-1} of

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 0 & 1 & 1 \\ -1 & -2 & 0 \end{bmatrix}.$$

Exercise #4:

(a) Let $f_{ij}(x)$ be n^2 differentiable functions in the interval (a, b) and let us consider the function $F(x) = \det(f_{ij}(x))$ for every x in (a, b). Show that the derivative $F'(x)$ is the sum of n determinants

$$F'(x) = \sum_{i=1}^{n} \det A_i(x),$$

where $A_i(x)$ is the matrix that we get by differentiating the functions on the i-th row of the matrix $(f_{ij}(x))$.

(b) Let $u_1(x), u_2(x), u_3(x)$, and $u_4(x)$ be differentiable functions in the interval (a, b) and let us consider the Wronskian matrix

$$W(x) = \begin{bmatrix} u_1(x) & u_2(x) & u_3(x) & u_4(x) \\ u_1'(x) & u_2'(x) & u_3'(x) & u_4'(x) \\ u_1''(x) & u_2''(x) & u_3''(x) & u_4''(x) \\ u_1'''(x) & u_2'''(x) & u_3'''(x) & u_4'''(x) \end{bmatrix}.$$

Show that the derivative of det $W(x)$ equals the determinant of the matrix that we obtain from $W(x)$ by differentiating all the elements of the last row, i.e.

$$(\det W(x))' = \det \begin{bmatrix} u_1(x) & u_2(x) & u_3(x) & u_4(x) \\ u_1'(x) & u_2'(x) & u_3'(x) & u_4'(x) \\ u_1''(x) & u_2''(x) & u_3''(x) & u_4''(x) \\ u_1^{lv}(x) & u_2^{lv}(x) & u_3^{lv}(x) & u_4^{lv}(x) \end{bmatrix}.$$

Exercise #5: For every pair of square matrices A and B, show that

$$\det \begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix} = \det A \det B.$$

Exercise #6: Compute the determinant of the 6×6 matrix

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 & -1 \end{bmatrix}.$$