MA561 – Modern Algebra Homework set # 1 The due date is on September 10 (Wednesday), 2008.

1. For each of the following pairs of integers a and b

* a = 20, b = 13* a = 792, b = 275* a = 11391, b = 1567

determine their greatest common divisor (a, b) and write (a, b) in the form ax + by for some integers x and y.

- **2.** Show that if the integer k divides the integers a and b then k divides as + bt for each pair of integers s and t.
- **3.** Let a, b be nonzero integers and let p be a prime such that p divides ab. Show that p divides either a or b.
- 4. Prove that if $a = a_n 10^n + a_{n-1} 10^{n-1} + \dots + a_1 10 + a_0$ is any positive integer then $a \equiv a_n + a_{n-1} + \dots + a_1 + a_0 \pmod{9}$.
- 5. Compute the remainder when 37^{100} is divides by 29.
- **6.** (a) Prove that the squares of the elements in $\mathbb{Z}/4\mathbb{Z}$ are just $\overline{0}$ and $\overline{1}$.
 - (b) Using part (a), prove for any integers a and b that a^2+b^2 never leaves a remainder of 3 when divided by 4.
- **7.** Prove that if $\overline{a}, \overline{b} \in (\mathbb{Z}/n\mathbb{Z})^{\times}$ then $\overline{a} \cdot \overline{b} \in (\mathbb{Z}/n\mathbb{Z})^{\times}$.
- 8. For the pair of integers a = 13 and n = 20, show that a is relatively prime to n and determine the multiplicative inverse of $\overline{13}$ in $\mathbb{Z}/20\mathbb{Z}$.