MA561 – Modern Algebra Homework set # 6 The due date is November 12 (Wednesday), 2008.

41. If H and K are finite subgroups of a group G then

$$|HK| = \frac{|H||K|}{|H \cap K|}.$$

42. (Frattini's argument) Let H be a normal subgroup of a finite group G and let P be a p-Sylow subgroup of H. Then $G = HN_G(P)$.

(Hint: apply the 2nd part of Sylow's theorem to the p-Sylow subgroups of H.)

- **43.** Let G be a group of order 99. Show that G is abelian.
- **44.** Let G be a group of order 168 and assume that $\{1\}$ and G are the only normal subgroups of G. How many elements of order 7 does G have?
- **45.** Prove that if |G| = 132 then G is not simple.
- **46.** Let G be a finite group and $H \leq G$. Show that there is a composition series of G one of whose terms is H.
- **47.** Prove that S_n is generated by $\{(i \ i+1) \mid 1 \le i \le n-1\}$.
- **48.** Show that $S_n = \langle (1 \ 2), (1 \ 2 \ 3 \ \cdots \ n) \rangle$ for all $n \ge 2$.