

MA 765 Homework 2

Due Friday, February 9

1. Let k be a field and let A and B be k -algebras. Show that $A \otimes_k B$ is the coproduct of A and B in the category of k -algebras. That is, for any k -algebra C and k -algebra homomorphisms $\varphi_A : A \rightarrow C$, $\varphi_B : B \rightarrow C$, there exists a unique k -algebra homomorphism $\varphi : A \otimes_k B \rightarrow C$, such that $\varphi_A = \varphi \circ (\text{id} \otimes 1)$ and $\varphi_B = \varphi \circ (1 \otimes \text{id})$.
2. Conclude that products exist in the category of affine varieties.
3. If X and Y are affine varieties, is the topology on $X \times Y$ the same as the product topology? (Hint: consider $X = Y = \mathbb{A}^1$.)