MA 665 EXERCISES 9

(1) Let $C.$ be a chain complex of R-modules. Show that the following are equivalent:
   (a) $C.$ is exact;
   (b) $C.$ is acyclic, that is $H_n(C.) = 0$ for every n;
   (c) The map from the all-zero chain complex to $C.$ is a quasi-isomorphism.

(2) Prove that a morphism $u : C. \to D.$ of chain complexes sends boundaries to boundaries and cycles to cycles. Conclude that H_n is a functor from the category of chain complexes of R-modules to the category of R-modules.

(3) Let f be a morphism of chain complexes. Show that if $\ker(f)$ and $\operatorname{coker}(f)$ are acylic, then f is a quasi-isomorphism. Is the converse true?