

CHIP FIRING EXERCISES 6

- (1) Let $g = 2k$ be even, let $m_j \gg g$ for all j , and let G be a chain of loops with torsion profile \vec{m} . Prove that the number of divisor classes on G of rank 1 and degree $k + 1$ is the k th Catalan number.
- (2) Let G be a chain of loops and let D be a divisor on G . Prove that the tableau associated to $K_G - D$ is the transpose of the tableau associate to D .
- (3) Let G be a graph and v a vertex. A *Weierstrass gap* for v is a nonnegative integer k such that $\text{rk}(kv) = \text{rk}((k - 1)v)$. Show that the Weierstrass partition $\lambda_{G,v}(0)$ admits the following description. Starting at the point $(0, g)$, construct a Catalan path by taking the k th step 1 unit up if k is a Weierstrass gap, and taking the k th step 1 unit to the right if k is not a Weierstrass gap. Then $\lambda_{G,v}(0)$ is the set of boxes lying above this Catalan path.