

## CHIP FIRING EXERCISES 8

- (1) Let  $\Gamma_1$  and  $\Gamma_2$  be metric graphs, and let  $\Gamma$  be the graph obtained by identifying one point of  $\Gamma_1$  with one point of  $\Gamma_2$ . Show that

$$\text{Jac}(\Gamma) \cong \text{Jac}(\Gamma_1) \times \text{Jac}(\Gamma_2).$$

- (2) Give an example of a graph  $G$  of genus 3 with the property that, for any choice of edge lengths, the corresponding metric graph possesses a divisor of degree 2 and rank 1.
- (3) Let  $\Gamma$  be a metric graph, and let  $G$  be a simple, bipartite model for  $\Gamma$ . Prove that the set of vertices of a single color in  $G$  is a rank determining set.