Course evaluations now open
→ They close on Sunday, May 1st at midnight

Final Exam on Wednesday, May 4th 10:30 AM - 12:30 PM
here in Chem-Phys 139

Exam is cumulative (roughly one-third will be on material covered since Exam 3)

9 multiple choice questions ← 60 points
3 short answer questions ← 40 points

What's on the Exam?

Intro
Basic functions, graphs, semi-log and double-log plots

Sequences
Definition, limits, recursively defined sequences, fixed points

Limits
Definition, properties, continuous functions, IVT, Sandwich Theorem

Derivatives
Definition, properties, sum rule, product rule, quotient rule, chain rule, related rates, implicit differentiation, log differentiation

Applications of Derivatives
Max/min, increasing/decreasing, concavity, MVT, optimization, limits of recursively defined sequences

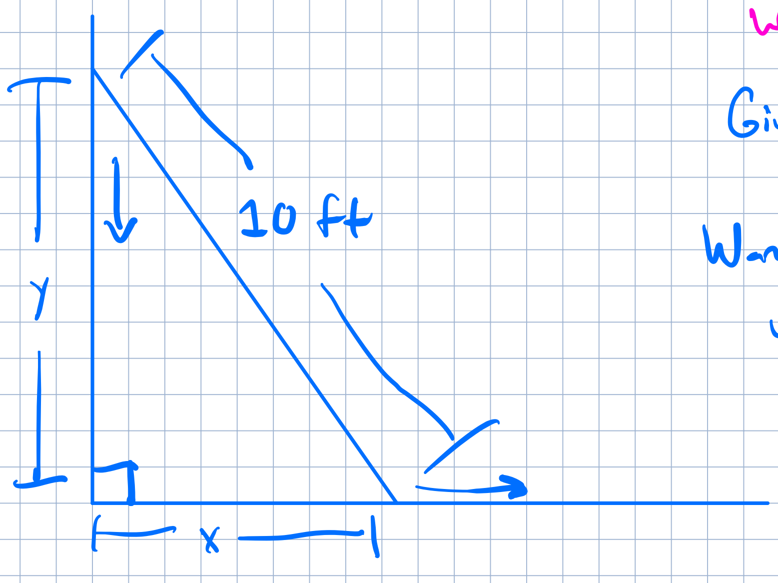
Integrals

Antiderivatives, Riemann sums, definition of the definite integral
Properties of the definite integral, FTOC

Related Rates

Ex: A 10 ft ladder leans against a wall.
If the top is falling at a rate of 2 ft/s,
how fast is the bottom moving away from the wall
when the top is 8 ft from the floor?

(1) Draw a picture



(2) What are we given?
What are we trying to find?

Given: $\frac{dy}{dt} = -2$ ft/s

Want to find: $\frac{dx}{dt}$
when $y = 8$ ft.

(3) Write an equation expressing the relationship between your variables

$$x^2 + y^2 = 10^2$$

$$x^2 + y^2 = 100$$

④ Take the derivative of both sides

$$\frac{d}{dt}\left[x^2+y^2\right] = \frac{d}{dt}\left[100\right]$$

$$2x\cdot\frac{dx}{dt} + 2y\cdot\frac{dy}{dt} = 0$$

⑤ Plug in

$$2x\,\frac{dx}{dt} + 2(8)\cdot(-2) = 0$$

$$2x\,\frac{dx}{dt} = 2\cdot 2\cdot 8 \qquad \frac{dx}{dt} = \frac{2\cdot 8}{x} = \frac{16}{x}$$

$$y = 8 \qquad x^2+y^2 = 10^2$$

$$\boxed{\frac{dx}{dt} = \frac{16}{6}\ \text{ft}/\text{s}}$$

$$x^2+8^2 = 10^2$$

$$x^2 = 10^2-8^2 = 100-64 = 36$$

$$x = 6$$

FTOC

Ex: Find $\frac{d}{dx}\left[\int_3^{x^3} \sec(t)\,dt\right]$

$$G(x) = \int_3^{x} \sec(t)\,dt$$

FTOC, part 1: $G'(x) = \sec(x)$

$$\int_3^{x^3} \sec(t)\,dt = G(x^3)$$

$$\frac{d}{dx}\left[G(x^3)\right] \overset{\text{chain rule}}{=} G'(x^3)\cdot\frac{d}{dx}\left[x^3\right]$$

$$= \boxed{\sec(x^3)\cdot 3x^2}$$

Ex: $\displaystyle\int_{\pi/6}^{\pi/4} \sec(x)\tan(x)\,dx$

$\sec(x)$ is an antiderivative of $\sec(x)\tan(x)$

By FTOC, part 2

$$\int_{\pi/6}^{\pi/4} \sec(x)\tan(x)\,dx = \sec(x)\Big|_{\pi/6}^{\pi/4}$$

$$= \sec(\pi/4) - \sec(\pi/6) = \frac{1}{\cos(\pi/4)} - \frac{1}{\cos(\pi/6)}$$

$$= \frac{1}{\frac{\sqrt{2}}{2}} - \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{2}} - \frac{2}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{\sqrt{6}} - \frac{2\sqrt{2}}{\sqrt{6}} = \frac{2(\sqrt{3}-\sqrt{2})}{\sqrt{6}}$$

Sandwich Theorem

Ex: Compute $\lim_{x \to 0} x^2 \cdot \sin\left(\frac{1}{x}\right)$

$$-1 \le \sin\left(\frac{1}{x}\right) \le 1$$

since $x^2 \geq 0$ for all x

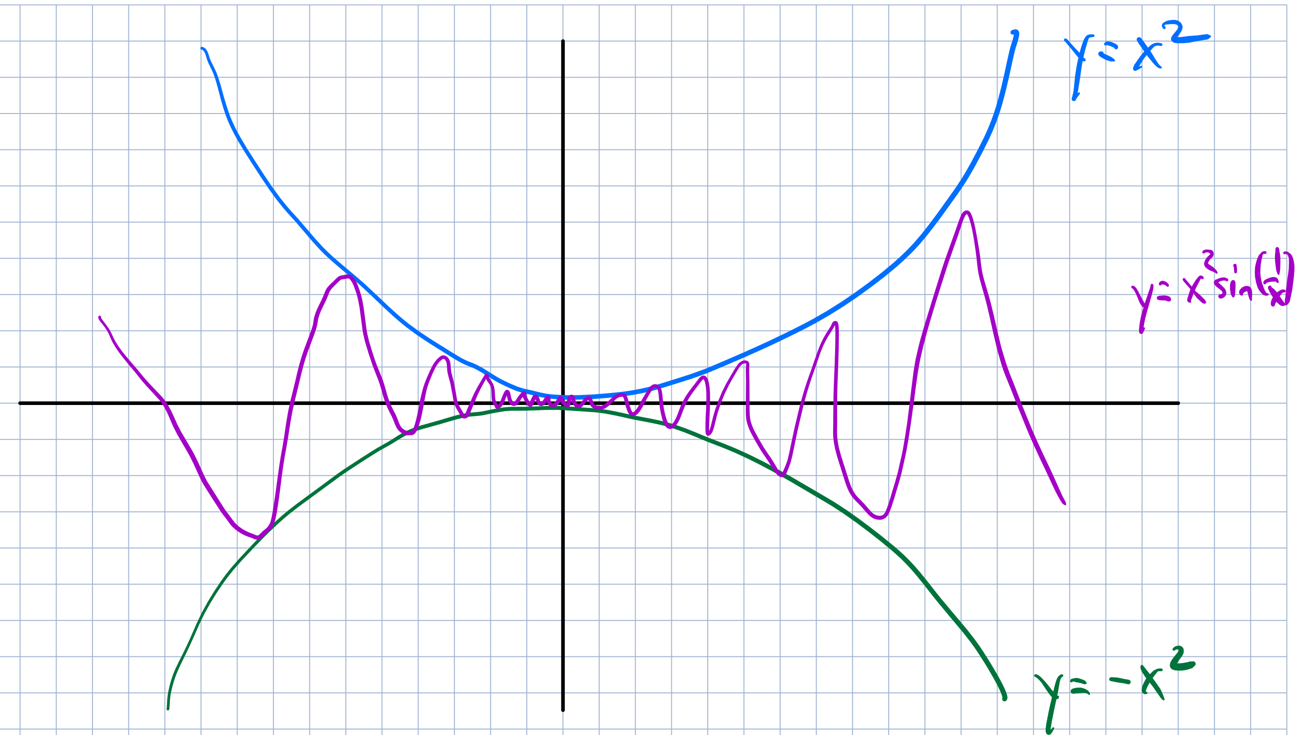
$$-x^2 \le x^2 \sin\left(\frac{1}{x}\right) \le x^2$$

$$\underbrace{\lim_{x \to 0} -x^2}_{= -0^2 = 0} \le \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \le \underbrace{\lim_{x \to 0} x^2}_{= 0^2 = 0}$$

because x^2 is a continuous function

$$0 \le \lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) \le 0$$

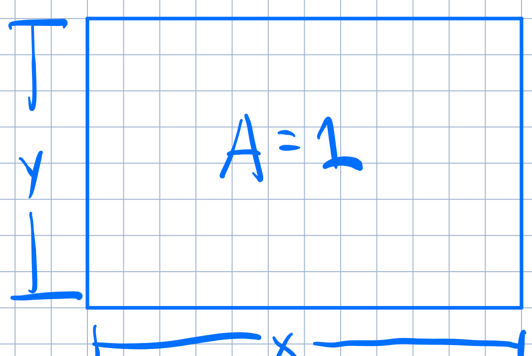
$$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$$



Optimization

Ex: Among all rectangles with area 1, find the one with the smallest perimeter.

(1) Draw a picture



(2) What function are you trying to optimize?

We want to minimize the <u>perimeter</u> P.

(3) Write this function as a function of a <u>single</u> variable.

$$P = 2x + 2y \qquad xy = 1 \qquad y = 1/x$$

$$P = 2x + \frac{2}{x}$$

(4) Take the derivative

$$\frac{dP}{dx} = \frac{d}{dx}\left[2x + \frac{2}{x}\right] = 2 - \frac{2}{x^2} = 0$$

$$2 = \frac{2}{x^2} \qquad 1 = \frac{1}{x^2} \qquad x^2 = 1$$

$$x = 1$$

$$y = \frac{1}{x} = \frac{1}{1} = 1. \longrightarrow y = 1$$

(5) Check that this is actually a min.

I'm going to use the second derivative test.

$$\frac{d^2P}{dx^2} = \frac{d}{dx}\left[2 - \frac{2}{x^2}\right]$$

$$= 0 - 2 \cdot \frac{d}{dx}\left[x^{-2}\right]$$

$$= -2 \cdot (-2) \cdot x^{-3} = \frac{4}{x^3}.$$

$\frac{4}{x^3} > 0$ if $x > 0$.

So the 2^{nd} derivative is always positive.

So P is always concave up.

So $x = 1$ is a local min by the 2^{nd} derivative test.

$y = P(x)$