János Bolyai

(1802–1860)

HIS LIFE AND WORK

In *The Odyssey*, Homer wrote "Few sons are the equals of their fathers; most fall short, all too few surpass them." None of the other mathematicians featured in this book were the sons of mathematicians—János Bolyai is the exception. His father, Farkas Wolfgang Bolyai (1775–1856), was a mathematician of some note. Both worked on the problem of Euclid's parallel postulate. The elder Bolyai tried and failed to prove the parallel postulate. When he learned that his son was pursuing the same problem, he advised his son to give it up, essentially giving the age-old advice "Do as I say, not as I do." János ignored his father's advice and ultimately devised a non-Euclidean geometry.

To understand the achievement of the son, we must first learn the work of the father. This essay is about both of them.

The Bolyais descended from the ancient Hungarian noble family of Bolya de Bolyai, which had been known for the heroic participation defending their homeland against the Turks as far back as the thirteenth and fourteenth centuries. The family had once been wealthy, but by the middle of the eighteenth century was no longer so. Farkas Wolgang Bolyai was born to Gáspar Bolyai and his wife Kryztina on the greatly diminished Bolyai estate in the village of Domáld near the town of

Marosvásárhely. At that time, Transylvania was part of the Kingdom of Hungary in the Austro-Hungarian Empire. Since the end of World War I, Transylvania has been part of Romania and Marosvásárhely has been known as Târgu Mureş.

Gáspar taught Farkas his first lessons until he enrolled Farkas, at the age of six, at the Calvinist Evangelical-Reformed School in the nearby city of Nagyszeven. The school's faculty immediately recognized Farkas's talents in language, music, and, of course, mathematics. At age twelve, his talents earned him the appointment as tutor to Simon Kemény, the eight-year-old son of a member of an important local Baron. This appointment enabled Farkas to pursue an education that his family probably could not have afforded. After finishing at the local school, Farkas and Simon enrolled at the Calvinist College in nearby Kolozsvár.

Hungary had won its freedom from the Ottoman Empire in 1699 only to find itself 90 years later under the thumb of the Hapsburg Austro-Hungarian Empire. Kolozsvár was a hotbed of liberalism and Hungarian nationalism when Farkas arrived in 1790. No doubt he was greatly influenced by the view that reason rather than faith lay at the heart of understanding the cosmos and improving the lot of humankind. In particular, it lay at the heart of developing a truly Hungarian culture.

When Farkas and Simon graduated in 1796, they chose not to pursue their education in Vienna, the Imperial capital. Instead, they matriculated first at the University in Jena in Prussia before moving on to the prestigious University of Göttingen. At Göttingen, they were exposed to the very newest results in mathematics and the sciences. Most importantly for Farkas, he had the good fortune to be at Göttingen at the same time as Karl Friedrich Gauss, the greatest mathematician of all time. They became close friends and corresponded with each other for the rest of their lives.

Farkas and Simon lived the good life in Göttingen for the two years it took them to complete their studies. Then Simon's father ran into financial difficulties, their credit evaporated, and Farkas could not even afford to return home. For a year he had to borrow and beg for money in order to eke out a meager existence until a friend from Hungary sent him enough money to pay off his debts and allow him to return home by foot.

Back in Kolozsvár he found employment as a tutor. Thinking his income sufficient to support a family, Farkas married Susanna von Árkos, the daughter of a local surgeon. A year later, on December 15, Susanna gave birth to their first born, a son named János. The financial demands of his new family soon forced Farkas to accede to his father's wishes that he return home to Marosvásárhely to take up a job teaching mathematics, physics, and chemistry at the local Calvinist College. Once again the job paid very little, but Farkas was able to supplement his income by writing and producing his own plays, running the college tavern, and even designing tiles and cast iron stoves.

János turned out to be Farkas and Susanna's only child to live past infancy. As his marriage to Susanna soured, Farkas turned his attention more and more to János. By the age of four, János was already able to point out the most prominent constellations in the night sky. He also demonstrated an easy facility with mathematics at an early age, being able to identify a number of geometric figures and even the sine function. He taught himself how to read by the age of five and by the age of seven was giving violin recitals! János was tutored at home until the age of nine. His father taught him mathematics and the best students at the local college tutored him in other subjects.

János entered the local college in 1812, shortly before his tenth birthday. Not surprisingly, he flew through his studies, often taking classes intended for students several years older. Noting that János mastered calculus by the age of thirteen, Farkas thought that his son could be best educated under Gauss's wing at Göttingen. No doubt this would have been the best plan for János. Unfortunately, Gauss rejected his old friend's suggestion and the Bolyais needed to turn elsewhere for János's continuing education.

When János graduated from the local college in 1818, neither the University at Vienna nor the one in Pest offered an attractive course in mathematics. Going to a foreign university was out of the question, being beyond Farkas's financial means. In the end, János decided to study military engineering at the Imperial Academy of Engineering in Vienna. He took only four years to complete a course that normally took seven. While in Vienna, János became close friends with Carl Szász who was enrolled for a law degree. They spent many hours together working on the parallel postulate.

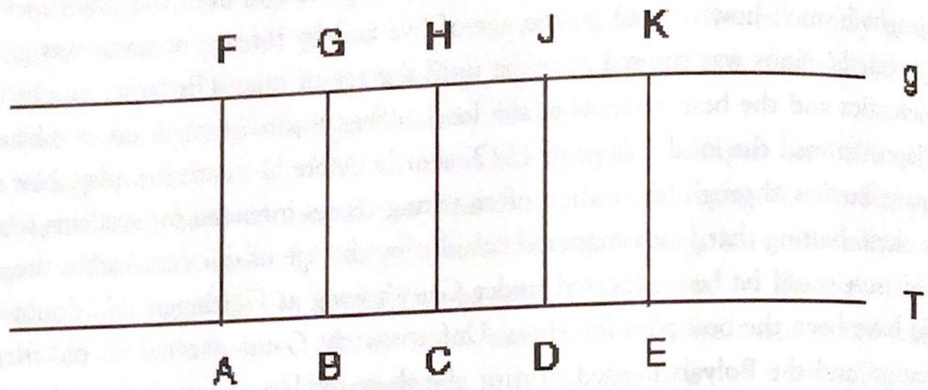
János began to work on Euclid's parallel postulate while he attended the Imperial Academy in spite of his father's warnings that he not do so.

> *You should not tempt the parallels in this way, I know this way until its end—I have also measured this bottomless night, I have lost in it every light, every joy of my life. . . . You should shy away from it as if from lewd intercourse, it can deprive you of all your leisure, your health, your peace of mind and your entire happiness—. This infinite darkness might perhaps absorb a thousand giant Newtonian towers, it will never be light on earth, and the miserable human race will never have something absolutely pure, not even geometry.*

Farkas had begun his own attacks on the parallel postulate while still in Göttingen and made his greatest progress while János was still a very young child.

Farkas pursued a path that had been trodden by many mathematicians before him trying to prove Euclid's parallel postulate. He attempted to prove that the "distance line" of a straight line is itself a straight line.

Farkas began by considering the segment *BD* and the segment *CH* perpendicular to it at *C*, its mid-point. He considers *T* to be the "geometrically movable" rigid figure *BCDH* (shaped like a *T* upside down) and moves *T* along me line *AE* in either direction $\pm\infty$. Farkas attempted to prove that the curve *g* described the point *H* is



a straight line. He first assumed that *g* is not a straight line. Noting that *g* would be symmetric with respect to the line including the segment *CH*, Farkas laboriously argued that *g* would intersect *CH* again at a point other than *H*.

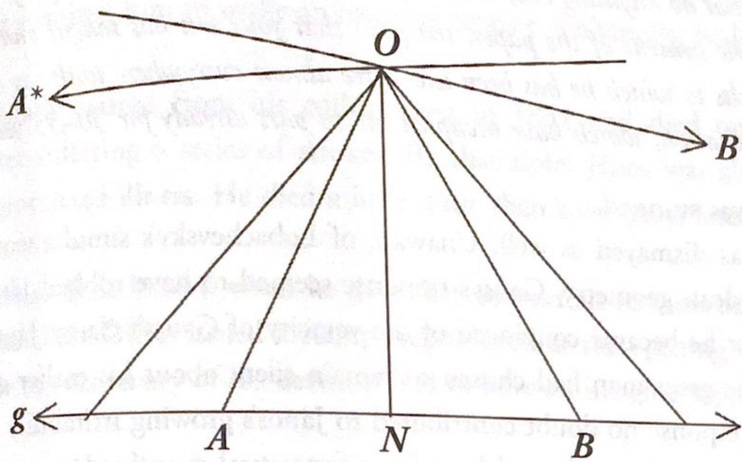
Farkas sent this proof to Gauss in late 1804. Gauss quickly spotted the error in Farkas's proof, namely the argument that *g* would intersect *CH* again at a point other than *H*, and wrote back informing his friend. It would later be shown that the assertion that *g* is a straight line is equivalent to Euclid's parallel postulate.

Gauss's response deflated Farkas so much that he gave up further attempts to prove the parallel postulate and spent the rest of his career trying to find simpler and more plausible equivalents to Euclid's parallel postulate. Thus, Farkas's 1820 warning to János should come as no surprise. It was simply a piece of fatherly advice. János paid it no heed and began his own attempts first to prove the parallel postulate and then to develop a non-Euclidean geometry.

Upon his graduation from the Imperial Engineering Academy in 1822, János was immediately commissioned as a sub-lieutenant in the Imperial army and posted to the city of Lvov, now part of the Ukraine. It was from Lvov that on November 3, 1823, János wrote to his father that "From nothing I have another, entirely new world."

János considers a line *g*, a point *O* not on *g*, and constructs the perpendicular from *O* to *g* that intersects *g* at the point *N*. He further supposes the points *A* and

B to lie on g on either side of N. Then, perhaps owing to his training as an engineer, or perhaps due to a suggestion of Szász, he *rotates* the half line OA counterclockwise about O and notes that there will be a position OA^* at which the half line *rebounds* from g and no longer intersects it. He considers the same rotation of OB clockwise about O rebounding from g at OB^*.



János then asks a very interesting question: are the half lines OA^* and OB^* extensions of each other or not? If they are, standard Euclidean geometry results. If they are not extensions of each other then a non-Euclidean geometry results in which the parallel postulate does not hold. Notice that in this case there is a positive angle between OA^* and OB^*. As a result the sum of the angles $<NOB^*$ and $<ONB$ is *less than* two right angles. From this János easily deduces that in this geometry the sum of the angles of a triangle is *less than* two right angles.

It would be more than a year until János could actually show his new geometry to his father. Farkas remained cautious at first. After all, he had once thought that he had made a significant breakthrough only to be disappointed severely. This mattered little to János who had little time to think about publishing his work owing to his frequent reassignments that took him to different parts of the eastern provinces of the empire. By 1831, however, Farkas had become enthusiastic about his son's work, describing it in his diary as an "original and great work, the greatest yet written by a Hungarian."

When János visited in 1831, Farkas urged János to allow him to include his *Science of Absolute Space* in the great mathematical compendium, the *Tentamen*, that he was publishing after a decade of effort. János agreed. Thinking of his son's accomplishment, Farkas sent a copy of János's essay to Gauss as soon as it was published.

After reading János's work Gauss wrote to one correspondent, "I regard this young geometer Bolyai as a genius of the first order." His response to Farkas was less complimentary and more matter of fact.

> *Now something about the work of your son. You will probably be shocked for a moment when I begin by saying I cannot praise it, but I cannot do anything else, since to praise it would be to praise myself. The whole content of the paper, the path that your son has taken, and the results to which he has been led, agree almost everywhere with my own meditation, which have occupied me in part already for 30–35 years.*

Farkas was stunned.

János was dismayed as well. Unaware of Lobachevsky's simultaneous discovery of non-Euclidean geometry, Gauss's response seemed to have robbed János of priority. Even after he became convinced of the veracity of Gauss's claim, János remained bitter that the great man had chosen to remain silent about his earlier discoveries.

Gauss's response no doubt contributed to János's growing irritability and declining health. Having been plagued by a fever for several months, János was forced to retire from the army in June 1833 at the age of 30. János first went to live with his father in Marosvásárhely. However, he did not get along with his step-mother Theresa, whom Farkas had married in 1824, three years after the death of János's mother.

After a year, János left for Domáld where he lived on the old Bolyai estate. It was there that he met Rosalie von Orbán who would become his common-law wife. His pension was so miniscule that he could not afford to make the deposit required for a proper marriage.

János continued to work on mathematics for the rest of his life, especially on the geometric representation of complex numbers. In 1837, the Jablonsky Society of Leipzig issued a call for papers on the topic. János was chagrined when he learned that his father had also submitted a paper to the society. Neither Bolyai's paper was published. Indeed, even though he wrote an additional 20,000 notebook pages, now housed in a library in Târgu Mureş, János published nothing but the *Science of Absolute Space* during his lifetime.

By 1846, Rosalie had borne János three children, two of whom survived. For some reason, perhaps wanting to be closer to his widowed father, János moved his family from Domáld back to Marosvásárhely that year. Moving closer to his father did not work out well. Relations between Farkas and János worsened.

One consequence of their proximity was that Farkas easily passed a copy of Lobachevsky's *Theory of Parallel Lines* on to János when a copy arrived in 1848. János studied Lobachevsky's work in great detail, recording his observations in a notebook labeled *Geometrical Examination.* At first he thought that Lobachevsky might be a fictional character made up by Gauss, perhaps. In the end, János recognized that he and Lobachevsky had independently discovered non-Euclidean geometry with the Russian being equally deserving of credit.

János and Rosalie finally had a proper marriage in 1849 after a change in the law no longer required him to make an onerous deposit. Unhappily, he left her three years later.

Farkas finally retired from his college post in 1851 and died on November 20, 1856 after suffering a series of strokes. By that time, János was already in the throes of a protracted illness. He died a little more than three years later on January 27, 1860.

János was so little known when he died that no portrait remains of him. When the Hungarian Post Office issued a stamp (reproduced at the opening of this chapter) to honor the centenary of his death, it had to have an imaginary portrait of him painted.

The *Science of Absolute Space* lay forgotten until Richard Baltzer discussed it along with Lobachevsky's *Theory of Parallel Lines* in the second edition of his *Elemente der Mathematik* in 1867. Further recognition came when the French mathematician Jules Hoüel published a French translation a year later. The ultimate tribute came at the end of the nineteenth century when the great French mathematician Henri Poincaré used the title "Bolyai-Lobachevsky geometry" to describe the non-Euclidean geometry these mathematicians had discovered coincidentally.