Chapter 2
Arithmetic in the Islamic World

1 The Decimal System

Muslim mathematicians were the first people to write numbers the way we do, and, although we are the heirs of the Greeks in geometry, the part of our legacy from the Muslim world is our arithmetic. This is true even if it was Hindu mathematicians in India, probably a few centuries before the rise of Islamic civilization, who began using a numeration system with these two characteristics:

1. The numbers from one to nine are represented by nine digits, all easily made by one or two strokes.
2. The rightmost digit of a numeral counts the number of units, and a unit in any place is ten of that to its right. Thus, the digit in the second place counts the number of tens, that in the third place the number of hundreds (which is ten tens), and so on. A special mark, the zero, is used to indicate that a given place is empty.

These two properties describe our present system of writing whole numbers, and we may summarize the above by saying the Hindus were the first people to use a cipherized, decimal, positional system, "Cipherized" means that the first nine numbers are represented by nine ciphers, or digits, instead of accumulating strokes as the Egyptians and Babylonians did, and "decimal" means that it is base 10. However, the Hindus did not extend this system to represent parts of the unit by decimal fractions, and, since it was the Muslims who first did so, they were the first people to represent numbers as we do. Quite properly, therefore, we call the system "Hindu–Arabic".

As to when the Hindus first began writing whole numbers according to this system, the available evidence shows that the system was not used by the great Indian astronomer Āryabhata (born in A.D. 476), but it was in use by the time of his pupil, Bhaskara I, around the year A.D. 520. (See Van der Waerden and Folkerts for more details.)

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News of the discovery spread, for, about 150 years later, Severus Sebokht, a bishop of the Nestorian Church (one of the several Christian faiths existing in the East at the time), wrote from his residence in Keneshra on the upper Euphrates river as follows:

> I will not say anything now of the science of the Hindus, who are not even Syrians, of their subtle discoveries in this science of astronomy, which are even more ingenious than those of the Greeks and Babylonians, and of the fluent method of their calculation, which surpasses words. I want to say only that it is done with nine signs. If those who believe that they have arrived at the limit of science because they speak Greek had known these things they would perhaps be convinced, even if a bit late, that there are others who know something, not only Greeks but also men of a different language.

It seems, then, that Christian scholars in the Middle East, writing only a few years after the great series of Arab conquests had begun, knew of Hindu numerals through their study of Hindu astronomy. The interest of Christian scholars in astronomy and calculation was, in the main, due to their need to be able to calculate the date of Easter, a problem that stimulated much of the Christian interest in the exact sciences during the early Middle Ages. It is not a trivial problem, because it requires the calculation of the date of the first new moon following the spring equinox. Even the great nineteenth-century mathematician and astronomer C.F. Gauss was not able to solve the problem completely, so it is no wonder that Severus Sebokht was delighted to find in Hindu sources a method of arithmetic that would make calculations easier.

We can perhaps explain the reference to the "nine signs" rather than the ten as follows: the zero (represented by a small circle) was not regarded as one of the digits of the system but simply a mark put in a place when it is empty, i.e., when no digit goes there. The idea that zero represents a number, just as any other digit does, is a modern notion, foreign to medieval thought. This is clearly shown in al-Khalili's auxiliary tables for certain combinations of trigonometric functions depending on two arguments, x and y. In the case of values of x and y that would produce a value outside the domain of the arcos function al-Khālilī writes "0 0," which can only mean 'no value' not zero degrees zero minutes.

With this evidence that the Hindu system of numeration had spread so far by the year A.D. 662, it may be surprising to learn that the earliest Arabic work we know of explaining the Hindu system is one written early in the ninth century whose title may be translated as *The Book of Addition and Subtraction According to the Hindu Calculation*. The author was Muḥammad ibn Mūsā al-Khwārizmī who, since he was born around the year A.D. 780, probably wrote his book after A.D. 800.

We mentioned in Chapter 1 that al-Khwārizmī, who was one of the earliest important Islamic scientists, came from Central Asia and was not an Arab. This was not unusual, for, by and large, in Islamic civilization it was not a man's place (or people) of origin, his native language, or (within limits) his religion that mattered, but his learning and his achievements in his chosen profession.

The question arises, however, where al-Khwārizmī learned of the Hindu arithmetic, given that his home was in a region far from where Bishop Sebokht learned

of Hindu numerals 150 years earlier. In the absence of printed books and modern methods of communication, the penetration of a discovery into a given region by no means implied its spread to adjacent regions. Thus al-Khwārizmī may have learned of Hindu numeration not in his native Khwārizm but in Baghdād, where, around 780, the visit of a delegation of scholars from Sind to the court of the Caliph al-Manṣūr led to the translation of Sanskrit astronomical works. Extant writings of al-Khwārizmī on astronomy show he was much influenced by Hindu methods, and it may be that it was from his study of Hindu astronomy that he learned of Hindu numerals.

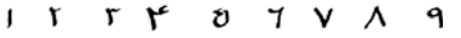
Whatever the line of transmission to al-Khwārizmī was, his work helped spread Hindu numeration both in the Islamic world and in the Latin West. Although this work has not survived in the Arabic original (doubtless because it was superseded by superior treatises later on), we possess a Latin translation, made in the twelfth century A.D. From the introduction to this we learn that the work treated all the arithmetic operations and not only addition and subtraction as the title might suggest. Evidently al-Khwārizmī's usage is parallel to the somewhat dated English description of a child who is studying arithmetic as "learning his sums."

2 Kūshyār's Arithmetic

2.1 Survey of The Arithmetic

As we have said, al-Khwārizmī's book on arithmetic is no longer extant in Arabic, and one of the earliest works on Hindu numeration whose Arabic text does exist was written by a man named Kūshyār b. Labbān, who was born in the region south of the Caspian Sea some 150 years after al-Khwārizmī wrote his book on arithmetic. Although Kūshyār was an accomplished astronomer, we know very little about his life, but despite this personal obscurity his works exerted some influence, and his treatise on arithmetic, whose title means *Principles of Hindu Reckoning*, became one of the main arithmetic textbooks in the Islamic world.

Kūshyār's concise treatise is a carefully written introduction to arithmetic, divided into two main parts. The first contains, after a brief introduction, nine sections on decimal arithmetic, beginning with "On Understanding the Forms of the Nine Numerals." In this the nine numerals are given in a form standard in the east, namely:



and the place-value system is explained. Zero is introduced as the symbol to be placed in a position "where there is no number." The Arabic word for zero, "ṣifr," comes from the verb "ṣafira" which means "to be empty or void" and it is the source, via French and Spanish, of our word "cipher". It is even the source, via