

MA 391 ASSIGNMENT # 5

Answers to problems may be handwritten.

- (1) In the cake-cutting procedure where one person cuts the cake and the other chooses which piece to eat, explain why it is more advantageous to be the cutter than the chooser. Why doesn't this contradict the fact that this procedure is envy-free?

- (2) Pam and Jose have a pizza that is one-third pepperoni, one-third mushrooms, and one-third sausage. Pam likes pepperoni and sausage equally well but does not like mushrooms. Jose likes pepperoni twice as much as sausage and likes mushrooms twice as much as pepperoni. They want to split the pizza by the divider/chooser method. After drawing straws, Jose is the divider.
 - (a) Draw a picture of the pizza.
 - (b) If Jose does not know Pam's preferences, how should Jose cut the pizza?
 - (c) Which piece should Pam choose?

- (3) Suppose that you and your roommate are renting a 2-bedroom apartment that costs \$1000 per month. The two bedrooms, the Big Bedroom and the Small Bedroom, are not equally desirable, but they are each okay. So you and your roommate decide that you will pay unequal rents depending on who takes the Big Bedroom and who takes the Small Bedroom.
 - (a) Why is there a rent so that you would be equally happy to live in the Big Bedroom for that rent or live in the Small Bedroom for \$1000 minus that rent?
 - (b) Why can you find prices for living in the apartment where both you and your roommate are happy to pay the price you each get?

- (4) In the previous exercise, suppose that you are willing to pay \$400 maximum for the Small Bedroom and \$600 maximum for the Big Bedroom, whereas your roommate is willing to pay \$300 maximum for the Small Bedroom and \$700 maximum for the Big Bedroom.
 - (a) Before deciding on actual rents, is it clear who should get the Small Bedroom and who should get the Big Bedroom? Why?
 - (b) Plot the points $(300, 700)$, $(400, 700)$, $(300, 600)$, $(400, 600)$ in the plane. Use two line segments to connect the points in an X shape. At what point do the two line segments intersect?
 - (c) Do these coordinates seem like reasonable rents for the two bedrooms? Why or why not?