Nos. 18-422, 18-726

IN THE
Supreme Court of the United States

ROBERT A. RUCHO, ET AL.,
Appellants,
v.
COMMON CAUSE, ET AL.,
Appellees.

On Appeal from the United States District Court for the Middle District of North Carolina

LINDA H. LAMONE, ET AL.,
Appellants,
v.
O. JOHN BENISEK, ET AL.,
Appellees.

On Appeal from the United States District Court for the District of Maryland

AMICUS BRIEF OF MATHEMATICIANS, LAW PROFESSORS, AND STUDENTS IN SUPPORT OF APPELLEES AND AFFIRMANCE

GUY-URIEL E. CHARLES
HARVARD LAW SCHOOL
1563 Massachusetts Avenue
Cambridge, MA 02138
(617) 998-1742

DEEPAK GUPTA
Counsel of Record
DANIEL WILF-TOWNSEND
GUPTA WESSLER PLLC
1900 L Street, NW, Suite 312
Washington, DC 20036
(202) 888-1741
deepak@guptawessler.com

Counsel for Amici Curiae

March 8, 2019

TABLE OF CONTENTS

TABLE OF AUTHORITIES

Cases

Other authorities

INTEREST OF *AMICI CURIAE*[1]

Amici are a group of mathematicians and law professors and a supporting team of students. Moon Duchin and the other *amici* mathematicians are members of a mathematical community that has worked to develop benchmarks for statistical and computational techniques that can be used to evaluate districting plans. Some have critically compared the work of other redistricting analysis teams, and some have developed their own algorithms and analytical techniques. Several *amici* have published in this area in popular forums in addition to their scholarly work.

Bridging the fields of law and mathematics, the *amici* have worked together in this brief to articulate a legal standard for adjudicating partisan-gerrymandering claims and to develop ideas for making the mathematics responsive to the needs of the Court. *Amici* write to inform the Court of the relevance and availability of reliable computational methods for evaluating these claims—methods that are illustrated using data in the North Carolina case now before the Court. A complete list of the *amici* is set forth in an addendum to this brief.

---

[1] No counsel for a party authored this brief in whole or in part and no person other than *amici* and their counsel made a monetary contribution to its preparation or submission. The parties' letters consenting to the filing of *amicus* briefs are on file with the Clerk.

SUMMARY OF ARGUMENT

The right to vote is more than just the right to cast a ballot. The Constitution protects an individual's right to an *undiluted* vote—a right to cast a vote that is free from being "impaired, lessened, diminished, diluted, and destroyed." *United States v. Saylor*, 322 U.S. 385, 388 (1944). The government violates the Constitution when it intentionally dilutes an individual's vote, whether on grounds of race, sex, geography, or partisanship.

An equality principle undergirds "one person, one vote": under the Constitution, the government unconstitutionally dilutes the right to vote when it draws district lines in a manner that arbitrarily distinguishes among voters. *See Gray v. Sanders*, 372 U.S. 368, 379 (1963). This equality principle is a judicially manageable standard. A plaintiff can prove a claim of vote dilution by showing that the manner in which the government drew the district lines diminished the equal weight, power, and value of the plaintiff's vote.

Because vote dilution violates basic constitutional rights, it is essential for courts to be able to detect and act on intentional dilution where it exists. But adjudicating the potential abuses of line-drawing for partisan advantage has bedeviled this Court for decades. Drawing on many years of experience in the fields of mathematics and computational science, this brief presents and illustrates a computational method that is designed to efficiently produce representative samples—in this application, by sampling alternative valid districting plans. Courts can then reasonably infer the presence of intentional discrimination by assessing a given district in the context of valid alternative plans. Such a comparison allows courts to see whether a challenged district falls within the wide range of plans that you would expect to

see from a neutral decision-maker, or is instead an outlier.

This brief thus offers a way forward that is based on reliable and well-established tools that have been used in science and industry for decades—in applications ranging from weapons development to weather prediction. The process of generating these large, diverse ensembles of possible districts also meets the demands of our constitutional framework by taking state-specific redistricting criteria and a state's unique political geography into account before flagging certain districts as outliers.

Unconstitutional vote dilution turns on the idea that the state has departed from a baseline of equal treatment. The method we describe here identifies that baseline by quantitatively determining a normal range for a given district's composition, making dilutive "packing and cracking" clearly visible to courts, legislatures, and the public when a district is an extreme outlier from that range. An outlier finding means that the design of the district is not explained by state-specific rules or political geography, but is far better explained by partisan motives.

In one of the two cases before the Court, the State of North Carolina expressed an explicit intent to produce a partisan outcome by sorting voters into districts on the basis of partisanship, thereby admitting to singling out voters to downgrade the value of their votes. Even without an explicit statement of intent, the methods described here demonstrate impermissible intent through extreme effects. The effects of unequal treatment of individual voters in multiple districts are so stark—and are so clearly shown not to be explained by

the rules and geography—that the only reasonable inference is one of intentional discrimination.

ARGUMENT

I. Claims of vote dilution on the basis of partisanship are justiciable.

The Constitution prohibits the state from intentionally infringing on an individual's right to vote, whether through denial, "debasement[,] or dilution." *Reynolds v. Sims*, 377 U.S. 533, 555 (1964). Vote dilution can take many forms—for example, stuffing a ballot box or intentionally mis-tabulating the votes. In the districting context it means that, to secure an outcome that it prefers, the state has drawn district lines in a way that intentionally discriminates among voters or treats voters differently. As a consequence of this intentional treatment, the state has minimized the weight, value, or power of certain votes. Vote dilution thus turns on the idea that the state has departed from a baseline of equal treatment and, by doing so, has intentionally diminished the weight, power, and value of an individual's vote. This Court has held for decades that the state violates the constitutional guarantee to an undiluted vote when it intentionally prefers some voters over others, whether based on race, sex, or geography. Now, this Court has the opportunity to affirm that the state likewise violates that constitutional guarantee when it prefers some voters based on their political affiliation or their political beliefs.

A. The Constitution supplies an equality principle that protects each individual from intentional vote dilution by the state.

The vote has long been regarded as among the most fundamental rights, and indeed as preservative of all other rights. *See, e.g.*, *Reynolds*, 377 U.S. at 562. This

Court has found "the right of suffrage can be denied by a debasement or dilution of the weight of a citizen's vote just as effectively as by wholly prohibiting the free exercise of the franchise." *Id.* at 555. The right is personal to the individual. *See*, *e.g., Gill v. Whitford*, 138 S. Ct. 1916, 1920 (2018) ("The right to vote is 'individual and personal in nature.'") (quoting *Reynolds*, 377 U.S. at 561).

While the "One Person, One Vote" principle can be conceptualized as a collective harm to principles of equal representation as a whole, *see Evenwel v. Abbott*, 136 S. Ct. 1120, 1131 (2016), the constitutional right to have an undiluted vote described by this Court can best be understood as an individual right that arises from an individual conception of injury. The state unconstitutionally diminishes this individual right when it draws districts to intentionally minimize or cancel out the votes of some voters, just as if it had stuffed the ballot box or intentionally miscounted votes to achieve its preferred outcome. *See id.* at 557–58. This constitutional violation occurs whether the state singles out specific voters based on race, geography, or political affiliation.

1. The Constitution protects an individual's right to an undiluted vote.

Every voter has an individual right to an undiluted vote. This Court has long recognized that "the right of suffrage can be denied by a debasement or dilution of the weight of a citizen's vote just as effectively as by wholly prohibiting the free exercise of the franchise." *Reynolds*, 377 U.S. at 555. A citizen's constitutional "right to vote free of arbitrary impairment by state action" encompasses a right to an election free "from dilution by a false tally" or "by a stuffing of the ballot box." *Baker v. Carr*, 369 U.S. 186, 208 (1962). Thus, the right to vote is not

merely the right to cast a ballot; it is the right to cast a ballot that is "honestly counted." *United States v. Saylor*, 322 U.S. 385, 388 (1944). The Constitution protects each voter's individual "right to have their expressions of choice given full value and effect by not having their votes impaired, lessened, diminished, diluted and destroyed." *Id.* at 386.

It is similarly well established that a state's drawing of its voting districts can dilute votes, thereby infringing on the individual right to vote. For example, in *Wesberry v. Sanders*, this Court struck down the State of Georgia's congressional apportionment plan for impermissibly diluting votes. 376 U.S. 1, 4 (1964). This Court expressly framed the constitutional violation in individual terms, explaining that when the state intentionally discriminates against some voters in the construction of voting districts, it "debas[es] the weight of the [plaintiffs'] votes." *Id.* This debasement "abridge[s] the right to vote for members of Congress guaranteed . . . by the United States Constitution." *Id.*; *see also Avery v. Midland Cty.*, 390 U.S. 474, 478 (1968) ("Every qualified resident, *Reynolds* determined, has the right to a ballot for election of state legislators of equal weight to the vote of every other resident, and that right is infringed when legislators are elected from districts of substantially unequal population."); *Fortson v. Dorsey*, 379 U.S. 433, 439 (1965) ("It might well be that . . . a multi-member constituency apportionment scheme, under the circumstances of a particular case, would operate to minimize or cancel out the voting strength of racial or political elements of the voting population.").

The fact that vote dilution in apportionment cases requires a comparison with the voting power of other voters does not change the individualized character of

the right recognized in this Court's early cases. As this Court articulated, "[s]imply stated, an individual's right to vote for state legislators is unconstitutionally impaired when its weight is in substantial fashion diluted when compared with votes of citizens living in other parts of the state." *Reynolds*, 377 U.S. at 568; *cf. id.* at 580 ("Citizens, not history or economic interests, cast votes. . . . Again, people, not land or trees or pastures, vote."). This Court's prior precedents are clear: the Constitution protects an individual voter from having their vote debased or diluted.

2. The government acts unconstitutionally when it intentionally dilutes an individual's vote.

The Constitution forbids the government and its officials from intentionally interfering with or impairing an individual's right to vote. *See United States v. Classic*, 313 U.S. 299 (1941); *see also Karcher v. Daggett*, 462 U.S. 725, 753 (1983) (Stevens, J., concurring) ("The path the Court has sometimes used to enter this political thicket is marked by the label 'intent.'"). This Court has upheld this prohibition on intentional interference in a variety of contexts, finding that a state unconstitutionally dilutes or impairs an individual's vote when it artificially inflates vote counts by stuffing ballot boxes, *see Classic*, 313 U.S. at 314 (citing *United States v. Mosley*, 238 U.S. 383 (1915)), arbitrarily refuses to count ballots from certain precincts, *see Reynolds*, 377 U.S. at 555 (citing *Saylor*, 322 U.S. at 386), redistricts with the purpose of disenfranchising a distinct segment of minority voters, *see Gomillion v. Lightfoot*, 364 U.S. 339, 347 (1960), restricts access to the primary when the primary is an instrumental mechanism of choice, *see Classic*, 313 U.S. at 314 (1941), and diminishes the strength of urban votes in

malapportioned systems, *see Reynolds*, 377 U.S. at 568. In each of these contexts, this Court reaffirmed that unconstitutional harm is done when the state manifests an intent to impair the participation of otherwise eligible voters.

The state cannot create a preferred class of voters. *See Gray*, 372 U.S. at 379–80 ("The concept of 'we the people' under the Constitution visualizes no preferred class of voters but equality among those who meet the basic qualifications."). A fundamental equality principle, grounded in the Equal Protection Clause, requires that "all who participate in the election are to have an equal vote—whatever their race, whatever their sex, whatever their occupation, whatever their income, and wherever their home may be in that geographical unit." *Id.* at 379.

The state impairs the participation of otherwise eligible voters when it creates a preferred class of voters. Thus, in *Gray*, this Court explained that a system in which a voter's ballot has more weight than a similarly situated voter in a neighboring county unconstitutionally discriminates between eligible voters. *Id.* at 381. The equality principle applies with the same force and with the same logic in the apportionment context and in all contexts in which the state delineates voting districts. For example, in *Wesberry*, the Court found that Georgia "grossly discriminate[d]" against the plaintiffs because its apportionment statute "contract[ed] the value of some votes and expand[ed] that of others." 376 U.S. at 7; *see also id.* at 17–18 ("Our Constitution leaves no room for classification of people in a way that unnecessarily abridges [the] right" to vote). Put simply, this Court has articulated and enforced a principle of equality among voters as a standard: all eligible voters have an equal right for their ballots to be cast, counted, and duly

registered free from state impairment, manipulation, and diminishment. And, as this Court has stated, the equality principle protects both the right of the voter to equal treatment and the right of the voter to equal representation. *Evenwel*, 126 S. Ct. at 1130.

Reynolds embraced this equality principle. There, this Court considered the constitutionality of the apportionment of the Alabama Legislature. *See Reynolds*, 377 U.S. at 545–46. This Court acknowledged that, together, *Gray* and *Wesberry* stand for the proposition that "in statewide and in congressional elections, one person's vote must be counted equally with those of all other voters in a State." *Id.* at 560. With this doctrinal foundation, the relevant question in *Reynolds* was "whether there are any constitutionally cognizable principles which would justify departures from the basic standard of equality among voters in the apportionment of seats in the legislature." *Id.* at 561. In other words, the fundamental equality principle *presumptively controlled*; the only question was whether there was a reason *not* to apply the fundamental equality principle to the issues presented in *Reynolds*. This Court found no such reason and affirmed the district court's order for the Alabama Legislature to be reapportioned. *See id.* at 587.

Since *Reynolds*, this Court has continually found attempted justifications to be insufficient to sustain population deviations that discriminate against similarly situated eligible voters based on their geographies—even though these claims do not implicate any suspect classifications. *See, e.g.*, *Chapman v. Meier*, 420 U.S. 1, 24 (1975); *Kirkpatrick v. Preisler*, 394 U.S. 526, 530–31 (1969). This Court has also affirmed that states cannot discriminate against voters under the guise of exercising "power wholly within the domain of state interest" in

their role facilitating elections. *Gomillion*, 364 U.S. at 347. Indeed, "[i]t is inconceivable that guarantees embedded in the Constitution of the United States may thus be manipulated out of existence." *Id.* at 345 (quoting *Frost & Frost Trucking Co. v. R.R. Comm'n*, 271 U.S. 583, 594 (1926)).

Because "the Constitution forbids 'sophisticated as well as simpleminded modes of discrimination,'" this Court looks beyond the statutory language and stated legislative purpose for evidence of unconstitutional intent to diminish the weight of certain individuals' votes. *Reynolds*, 377 U.S. at 562–64 (quoting *Lane v. Wilson*, 307 U.S. 268, 275 (1939)). Specifically, the real-world effects of state decisions can be used to discern unconstitutional intent. In *Gomillion*, this Court found an analysis of the effects of redistricting plan made a demonstration, "tantamount for all practical purposes to a mathematical demonstration, that the legislation is solely concerned with segregating white and colored voters by fencing Negro citizens out of town so as to deprive them of their pre-existing municipal vote." 364 U.S. at 341; *see also Miller v. Johnson*, 515 U.S. 900, 916 (1995) (explaining that intent can be shown "either through circumstantial evidence of a district's shape and demographics or more direct evidence going to legislative purpose").

B. The equality principle extends to cases in which the government intentionally dilutes votes based on partisanship.

As this Court has repeated, "in situations involving elections, the States are required to insure that each person's vote count as much, insofar as it is practicable, as any other person's." *Hadley v. Junior Coll. Dist.*, 397 U.S. 50, 54 (1970). Insuring that each person's vote

counts equally means that the state cannot dilute the power of votes by making distinctions—such as those based upon sex, geography, or race—among voters. This standard, the equality principle, applies to cases in which the government draws voting districts that intentionally discriminate against voters because of their political affiliation just as it applies to intentional discrimination against voters based on race, sex, or geography. When this Court is confronted with districts that show clear intent to dilute the power of voters based on political belief or party affiliation—as it is in this case—it must conclude that these cases are justiciable, consistent with this Court's prior precedents.

Vote dilution based on voters' political beliefs is structurally no different than vote dilution on other bases, such as race or geography, that this Court has found justiciable. As this Court explained in *Baker*, "if 'discrimination is sufficiently shown, the right to relief under the equal protection clause is not diminished by the fact that the discrimination relates to political rights.'" 369 U.S. at 209–10 (quoting *Snowden v. Hughes*, 321 U.S. 1, 11 (1944)). *See also Rogers v. Lodge*, 458 U.S. 613 (1982) (reaffirming the justiciability of race-based vote dilution claims and deciding one such claim on the merits); *White v. Regester*, 412 U.S. 755, 766–67 (1973) (same); *Whitcomb v. Chavis*, 403 U.S. 124, 143 (1971) (same). This Court has stated that dilution based on sex would not be "allowable." *Gray*, 372 U.S. at 379. And this Court has repeatedly found justiciable claims of vote dilution based on discrimination by geography, in which urban voters were systematically assigned to larger districts, amounting to a lesser weight being afforded to their votes. *See, e.g.*, *id.*; *Reynolds*, 377 U.S. at 562–64. In *Gray*, this Court explained that the justiciability of geography-based vote dilution claims follows logically from the

justiciability of race- and sex-based vote dilution claims. When "none could successfully contend" that race- and sex-based vote dilution "was allowable," "[h]ow then can one person be given twice or 10 times the voting power of another person in a statewide election merely because he lives in a rural area or because he lives in the smallest rural county?" *Gray*, 372 U.S. at 379.

The same reasoning applies in cases alleging vote dilution on the basis of partisanship. There is no principled basis on which the merits of claims alleging party-based dilution can be distinguished from those of claims alleging intentional dilution based on race, sex, and geography. The justiciability of party-based dilution claims flows ineluctably from geography-based and race-based dilution claims. If the state impermissibly impairs the right to a vote when it dilutes the votes of those who reside in the city, *id.* at 379, draws racial gerrymanders, *see id.*, stuffs ballot boxes, *Saylor*, 322 U.S. at 389, or discriminates based on sex, *Gray*, 372 U.S. at 379, then it is difficult to see why it would not also be an impermissible impairment for the state to discriminate based on political affiliation.

Just as it would violate the Constitution if the state decided to give double votes to urban voters or to White voters, but only single votes to suburban voters or to Black voters, it would also violate the Constitution if the state gave double votes to some voters and only single votes to others on the basis of partisanship. And just as it would violate the Constitution for the state to dilute the votes cast by individuals because they reside in the city or because they are Black, it also violates the Constitution when the state dilutes votes because the voters are associated with Republicans or with Democrats. In all these intentional vote dilution cases, this Court must

conduct the *Reynolds* inquiry, and determine whether there is any constitutionally valid justification for treating voters differently, and thus violating "the basic standard of equality among voters." *Reynolds*, 377 U.S. at 561.

Here, this Court should affirm that there is no such constitutionally valid justification from departing from presumptive voter equality. *Baker* protected the "right to a vote free of arbitrary impairment by state action." 369 U.S. at 208. No person may be given greater voting power merely because of her race, her sex, or where she lives. Surely, then, she may not be given greater voting power because she is a Democrat. This is especially so given that the First Amendment protects individuals from state discrimination on the basis of their political beliefs.

For example, the government may not determine access to government employment or benefits on partisan grounds, except in certain cases in which party affiliation is relevant to the office. *See Rutan v. Republican Party of Ill.*, 497 U.S. 62, 74–75 (1990); *Elrod v. Burns*, 427 U.S. 347, 359–60 (1976) (opinion of Brennan, J.). Similarly, membership in a particular political organization cannot be the basis for termination from the civil service. *See Keyishian v. Board of Regents*, 385 U.S. 589, 607–08 (1967). By denying a benefit to certain persons because of their partisan affiliation or their political beliefs, their "exercise of those freedoms would in effect be penalized and inhibited. This would allow the government to 'produce a result which (it) could not command directly.'" *Perry v. Sindermann*, 408 U.S. 593, 597 (1972) (quoting *Speiser v. Randall*, 357 U.S. 513, 526 (1958)). So too here: by disfavoring individuals based on their political affiliation, the state not only treats them

unequally in the exercise of the franchise, but it also penalizes and inhibits their First Amendment freedoms—a result it could not command directly.

* * *

In short, there is no constitutional principle that can explain why intentional, selective treatment based on individuals' partisanship should be adjudicated differently from intentional dilution on other grounds—race, sex, or geography. This Court has explained that "[t]he concept of 'we the people' under the Constitution visualizes no preferred class of voters but equality among those who meet the basic qualifications." *Gray*, 372 U.S. at 380. Moreover, there *is* a constitutional principle, embodied in the First Amendment, for why partisan association and beliefs should be protected. As *Wesberry* put it, "the right to vote is too important in our free society to be stripped of judicial protection." 376 U.S. at 7. *Reynolds* reaffirmed that, "[e]specially since the right to exercise the franchise in a free and unimpaired manner is preservative of other basic civic and political rights, any alleged infringement of the right of citizens to vote must be carefully and meticulously scrutinized." 377 U.S. at 562. The equality standard is judicially manageable, as reflected by this Court's prior precedents. Thus, this Court can, and should, conclude that these cases of intentional vote dilution on the basis of partisanship are justiciable, and that individuals deserve protection from this form of vote dilution just as for other forms already held to be unconstitutional.

II. Claims of vote dilution on the basis of partisanship can be evaluated by a reliable and well-established computational method.

In this section, we describe a powerful method to evaluate the districts in contested plans, setting a high

bar to distinguish extreme outliers from those within the range of reasonable outcomes for that state. Unconstitutional vote dilution can be proved by showing that the manner in which the government drew the lines departed from a baseline of equal treatment by diminishing the weight, power, and value of an individual's vote. The district court in the North Carolina case framed matters similarly, observing that "there needs to be a baseline from which to measure to what degree a districting plan drawn on the basis of partisan favoritism deviates from the universe of 'fair and effective' plans." *Common Cause v. Rucho*, 318 F. Supp. 3d 777, 876 n.33 (M.D.N.C. 2018) (citing *Vieth v. Jubelirer*, 541 U.S. 267, 307 (2004) (Kennedy, J., concurring)). Thus, a key to adjudicating cases of partisan gerrymandering is the determination of this baseline: we must have a reliable method to distinguish a normal, neutral, or non-gerrymandered district from an intentionally abusive, gerrymandered, dilutive district. After all, a great many legally valid congressional districting plans exist for North Carolina—not simulations, but true alternative plans—and they come in enormous variety. We must therefore create a benchmark understanding of neutral districting plans in a state-specific setting. Once we have such a benchmark, we can compare it to the challenged districting plan to determine whether, in light of the evidence, an intent to discriminate is the best explanation for a district's design.

The North Carolina case provides a demonstration of how such a model can work in practice. Here, there is no doubt about the intent behind the districting plan. David Lewis, the Republican official who led the redistricting process, admitted that his caucus had selected a map designed to produce a 10-3 split "because I do not believe it's possible to draw a map with 11 Republicans and two

Democrats." Compl. at 13, *Common Cause v. Rucho*, No. 1:16-CV-1016 (M.D.N.C. 2017), 2017 WL 3981300. Representative Lewis elaborated: "I think electing Republicans is better than electing Democrats, so I drew this map to help foster what I think is better for the country." *Id.* Months later, the map produced precisely the expected 10–3 outcome.

The method outlined below demonstrates how, even without such a helpfully clear statement of intent, such discrimination could be reasonably inferred. A simple forensic analysis reveals that six of the thirteen districts are extreme outliers in their partisan composition, both in the North Carolina Congressional Plan enacted in 2012 and in its 2016 replacement that has been held unconstitutional by the district court in the current case (the "2012 Plan" and "2016 Plan," respectively).

A. A reliable and well-established computational method provides a baseline that enables the identification of outliers.

For many decades, scientists, mathematicians, technologists, and government officials have used a technique known as *Markov chain Monte Carlo* ("MCMC") for prediction, modeling, and analysis of large data sets. MCMC gets its name from combining a class of algorithms known as Monte Carlo methods with a probabilistic process called a Markov chain. It has a sixty-year track record of success in studying configurations of complex systems, and has been famously applied in settings from weapons development to weather prediction to finance to number theory to quantum mechanics. *See generally* Persi Diaconis, *The Markov Chain Monte Carlo Revolution*, Bulletin of the American Mathematical Society, April 2009, at 179; Peter L. Galison, *Image & Logic: A Material Culture of Micro-*

physics (1997). For its use in redistricting, see Br. for Eric Lander, as Amici Curiae Supporting Appellees, *Gill v. Whitford*, 138 S. Ct. 1916 (2018) (No. 16-1161).

MCMC permits us to carry out a comparative analysis of districting plans by generating a large and diverse sample of districting plans—tens of thousands, millions, or billions can be efficiently generated on a standard laptop. The search can be restricted to plans that comply with a given state's districting laws, and hold constant the state's geography and voting patterns. These collections, or "ensembles," of plans provide a baseline understanding of the range of possible legally valid plans, which enables us to compare any given plan with that range and flag extreme outliers. We can refer to this assessment technique as "the method of ensembles."

Scientific consensus in the mathematics and statistics community increasingly endorses this approach to the problem of discriminatory redistricting. Many technical teams are now using this method independently of each other to understand redistricting in the United States. These teams each set up their own algorithms, running chains of map generation until they achieve high confidence that the ensemble is mathematically representative of the full universe of possibilities and provides robust, reliable results.

In the North Carolina case, plaintiff's expert Jonathan Mattingly adopted this approach and created an ensemble of 24,518 maps using a popular MCMC variant known as simulated annealing. *See districtingDataRepository*, Gitlab Community Edition, https://bit.ly/2HrB9mj. Other groups have similarly created redistricting ensembles using their own MCMC approaches. *See mggg*, Github, https://bit.ly/2TEHXTS; *redist*, Github, https://bit.ly/2EI1p8M; https://bit.ly/2XKe1F9.

Significantly, even though MCMC approaches vary, the results in the current case can be replicated. For instance, *amici* were able to compare Mattingly's ensemble to one that we prepared independently with an entirely different MCMC technique called tree-based recombination ("ReCom"), producing remarkably consonant results. To make the box-and-whisker plots shown below in Figure 1, the resulting maps were laid over Senate 2016 voting patterns to benchmark the expected partisan composition for each district in North Carolina—a similar plot could be made with respect to any other electoral data. The boxes show the middle 50 percent of plans, and the whiskers bracket the first to 99th percentile. Crucially, the two ensembles flag exactly the same districts as extreme outliers in both the 2012 Plan and 2016 Plan. This outlier finding will be illustrated for Districts #10 and #11 below (see Fig 2 and 3).

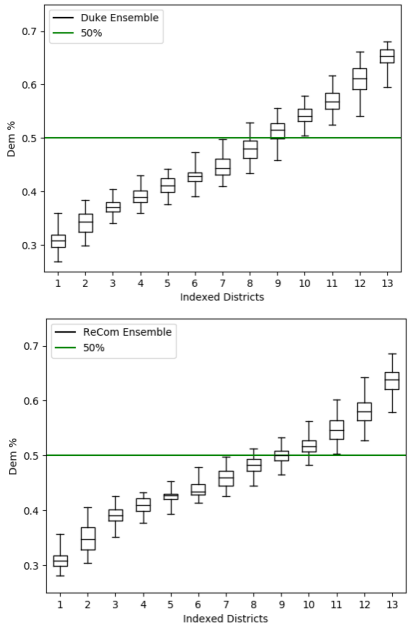

Figure 1.

These identical outlier findings, despite different MCMC designs and ensemble sizes, demonstrate the robustness of the approach. They also demonstrate that the method of ensembles is reliable and replicable, bolstering the plausibility of using outlier analysis as a legal tool.

B. This method takes into account the legal and political landscape of each state.

Individual states can and do set their own districting rules and criteria. This presents us with an explicit *ex ante* framework in each state, containing both federal rules, like equal population and the Voting Rights Act, and state-specific rules—Arizona's priority for competitive districts, Iowa's requirement that counties be preserved whole, Colorado's guidance to minimize the total perimeter of the districts, and so on.

In addition to laws governing districting, each state also has idiosyncratic physical and human geography: The Black population is concentrated in the Delta region of Mississippi; Democratic votes are concentrated in cities in Pennsylvania but in rural areas in Alaska; Iowa is a nearly-perfect grid of nearly-square counties; and so on.

The political geography, in particular, matters enormously for the determination of district behavior. For instance, if every household in a state casts three Republican votes and two Democratic votes, then as long as district lines do not split a house, every single district—no matter how its lines are placed—must necessarily reflect this 3–2 Republican tilt. This forces 100 percent of the representation to be Republican, irrespective of districting choices, which should clearly not support a finding of unconstitutional vote dilution. On the other hand, if the same 40 percent share of Demo-

cratic votes fell into several enclaves, then substantial Democratic representation would be normal and expected, and a 100 percent Republican outcome should be flagged as an outlier.

With MCMC, it is easily demonstrable that legal and political geography work together to create measurably different baselines in some states than in others. In other words, the outcomes of the districting process will and should vary a great deal from state to state, even without any discriminatory intent.

It is worth emphasizing that the technique we describe here is a *method*—not a new score of partisan skew. The method of ensembles does not produce a number or score. Instead, it generates a neutral baseline that can be used to interpret scores for a challenged district plan. The examples in the figures shown here demonstrate the method with respect to vote share, but the method applies equally to other measures or metrics of partisan skew. Thus, ensemble analysis can be flexibly applied to any one or several measures of partisanship that are specified in state rules or doctrinal criteria. A state can certainly name preferred scores of partisan fairness, as some are starting to do—Utah's new law cites partisan symmetry measures, while Missouri's uses efficiency gap. *See generally* Richard H. Pildes, *Redistricting Reform and the 2018 Elections*, Harvard Law Review Blog (Oct. 26, 2018), https://bit.ly/2TC1tAq. These methods then generate legally valid plans to benchmark state-specific reasonable values for each metric, rather than relying on universal *a priori* presumptions about ideals and thresholds. Ensembles only provide benchmarks and baselines once metrics have been selected.

C. Unconstitutional vote dilution can be identified in specific districts.

By identifying some districts as extreme outliers, the method of ensembles makes dilutive "packing and cracking" clearly visible to courts, legislatures, and the public. Packing and cracking are familiar concepts in the history of redistricting. Packed districts have elevated vote levels for a disfavored group, far beyond 50 percent and far from the baseline of the sample. Cracked districts have depressed vote share for the disfavored group, usually well under 50 percent even though the bulk of the ensemble is over that level. Because of asymmetrical geographical patterns, it may well be that very high or very low vote share in a district is completely expected without any discriminatory intent, so neither packing nor cracking is a manageable or usable concept without a method of distinguishing outliers from expected outcomes.

Exploring further the partisan composition data displayed above, we can sort the districts in order of their Democratic share and obtain a clear finding of extreme packing and cracking. The figure below uses the cracked 13th District from the 2016 Plan and the packed 4th District from that plan to illustrate how extreme outliers are made manifest by comparison with an ensemble. (These districts appear as #10 and #11, respectively, when sorted by Democratic vote share as in Figure 1.)

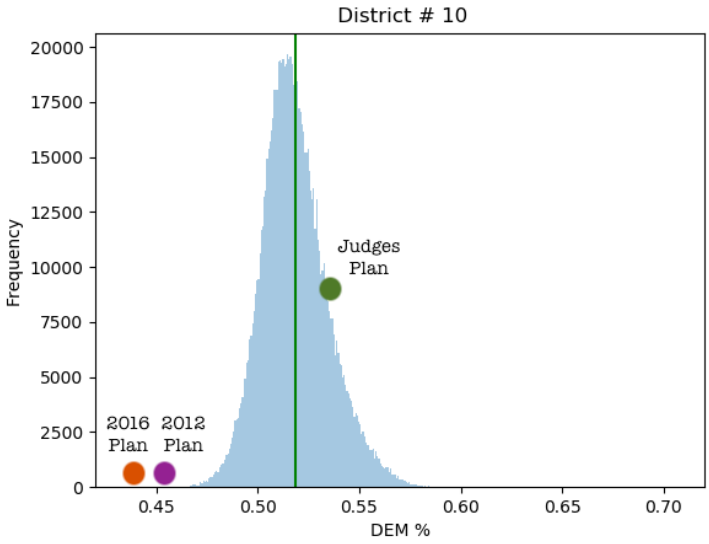

Figure 2

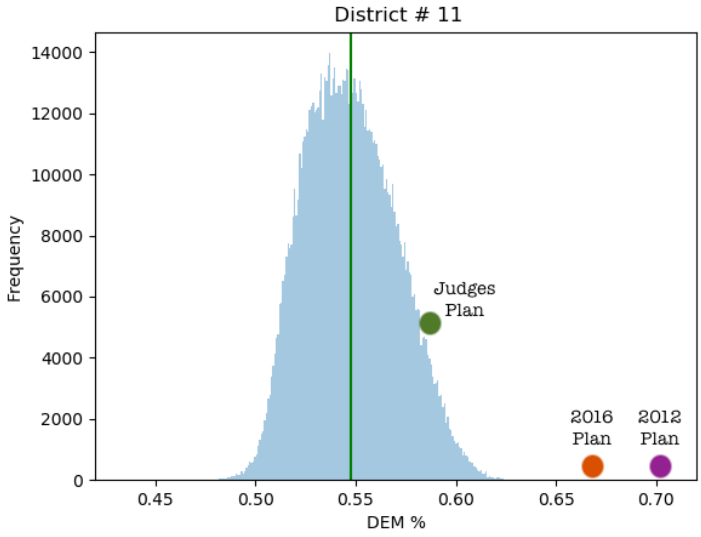

Figure 3

The extreme outlier status of these districts in both the 2012 Plan and the 2016 Plan is clearly visible here,

further thrown into relief by comparison with the corresponding districts in a plan made by a bipartisan panel of retired judges ("Judges' Plan"). Karen Kemp, *Nonpartisan Redistricting Panel Reveals Unofficial NC Congressional Voting Map*, Duke Sanford School of Public Policy (Aug. 29, 2016), https://bit.ly/2HjYyG8. Intentional minimization of the weight of votes for those districts' inhabitants is the clear inference from such extreme outlying effects.

The same effects may be observed if the comparison ensemble is sorted geographically rather than by party share, which we have done by designating a central Charlotte precinct and its district in the 2016 plan. Partisan composition in that district can then be compared to corresponding districts in the ensemble, averaged over their respective precincts. The figure below shows that the 2016 Plan clearly packs Charlotte voters. The 2012 Plan is not depicted in the figure because its packing is so pronounced that it is out of range of the plot. As with the other findings described in this section, Charlotte's outlier status is a matter of complete agreement between the Mattingly and the ReCom ensembles.

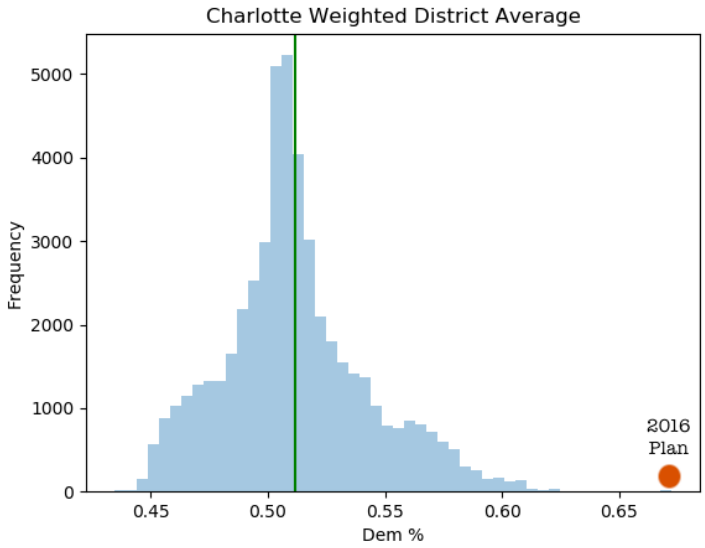

Figure 4

In sum, individual voters are harmed by living in districts that are intentionally designed to instrumentalize rather than to fully register their votes. When district design serves to effectively preordain an outcome far from the neutral baseline, it does not properly weight the votes in those districts.

Consider a scenario in which the North Carolina legislature decides to design the districts *after* a vote has been conducted, with knowledge of how votes were cast in each precinct. Having decided to concede three districts to Democrats, it can then sort the heavily Democratic precincts into those designated districts, clearly serving to diminish the weight, power, and value of those votes. The debasement of vote weight is as clear as in ballot-stuffing scenarios. The difficulty is to distinguish intentional manipulation by the state from the natural patterns of higher and lower concentration present in the vote distribution itself.

A plan composed of districts that fall far from the baseline constructed by the state's rules and geography, and in a pattern that benefits the controlling party, is not rationally explained by the rules and geography. Absent plausible justification by permissible principles, the reasonable inference is one of intentional discrimination.

D. This method is limited in scope.

A method that flags extreme outliers does not infringe on states' ability to set the rules and add districting criteria, nor on their latitude to select a plan that broadly comports with those criteria. We emphasize that the use of the method of ensembles for districting is proposed as an assessment technique, not proposed for optimization or map selection. This will never amount to usurping the state's authority to select a plan, because billions of substantially different plans remain viable, under any conception of outlier. This method does not choose a winner from among the abundance of options. This balances between state prerogatives and constitutional principles.

Importantly, even some districts or plans that look gerrymandered on their face will not be flagged as outliers by this method. For instance, Massachusetts had ten House seats in the 2000–2010 census cycle, and in that period, a Republican share of 30-37% was typical in statewide races. However, not a single Republican was elected to Congress in the five races in that cycle. This may seem to provide a cause of action for a potential claim. What an ensemble analysis clearly shows, however, is that for most elections in that cycle, no valid districting plan *whatsoever* will have even a single Republican-favoring district. Moon Duchin et al., *Locating the Representational Baseline: Republicans in Massachusetts*, MGGG (Oct. 22, 2019), https://bit.ly/

2SPW2cL. Not only a majority of possible plans, but indeed *every single* possible plan, produces a completely Democratic delegation. This is because Massachusetts at the time behaved much like the hypothetical state in which every household has the same voting preference: the statewide share of Republican votes is nearly replicated in every town and even precinct around the state. Thus the method of ensembles contradicts the *prima facie* suggestion of a gerrymander. This example also demonstrates that the method of ensembles does not covertly enforce a proportionality standard, but instead defers to the consequences of the state's rules and political geography.

CONCLUSION

The right to vote is personal and individual. That right is violated when a state singles out voters—in this case, because of their political beliefs—and acts with an intention to dilute their votes. The districting process by its very nature draws lines, thereby treating some voters differently from others. Thus, the process of distinguishing between this permissible line drawing and intentional, unconstitutional vote dilution is not always straightforward. The methods described here, however, demonstrate that these distinctions can be reliably made and thus that these claims can be justiciable. By flagging some districts as extreme outliers, these methods can make clearly visible to courts, legislatures, and the public when there is no rational justification for the drawing of a district and the only reasonable inference is one of intentional discrimination.

Respectfully submitted,

DEEPAK GUPTA
Counsel of Record
DANIEL WILF-TOWNSEND
GUPTA WESSLER PLLC
1900 L Street, NW, Suite 312
Washington, DC 20036
(202) 888-1741
deepak@guptawessler.com

GUY-URIEL E. CHARLES
HARVARD LAW SCHOOL
1563 Massachusetts Avenue
Cambridge, MA 02138
(617) 998-1742

March 8, 2019 *Counsel for Amici Curiae*

ADDENDUM: IDENTITY OF *AMICI CURIAE*

Moon Duchin is an Associate Professor of Mathematics at Tufts University and a Senior Fellow in the Tisch College of Civic Life.

Jeanne Clelland is a Professor of Mathematics at the University of Colorado Boulder.

Daryl DeFord is a postdoctoral associate in the Geometric Data Processing Group at the Massachusetts Institute of Technology.

Jordan Ellenberg is the John D. MacArthur Professor of Mathematics at the University of Wisconsin-Madison.

Tyler Jarvis is a Professor of Mathematics at Brigham Young University.

Nestor Guillen is an Assistant Professor of Mathematics at the University of Massachusetts at Amherst.

Dmitriy Morozov is a Staff Scientist in the Computational Research Division at the Lawrence Berkeley National Laboratory.

Elchanan Mossel is a Professor of Mathematics and core faculty of the Institute for Data, Systems and Society at the Massachusetts Institute of Technology.

Dana Randall is the ADVANCE Professor of Computing and the Co-Executive Director of the Institute for Data Engineering and Science at the Georgia Institute of Technology.

Justin Solomon is an Assistant Professor of Electrical Engineering & Computer Science at the Massachusetts Institute of Technology and leads the Geometric Data Processing Group.

Ari Stern is an Associate Professor of Mathematics and Statistics at Washington University in St. Louis.

Law Professors

Guy-Uriel Charles is the Edward and Ellen Schwarzman Professor of Law at Duke Law School, the Bennett Boskey Visiting Professor of Law at Harvard Law School, and the co-director of the Duke Law Center on Law, Race and Politics.

Luis Fuentes-Rohwer is a Professor of Law and Harry T. Ice Faculty Fellow at Maurer School of Law, Indiana University Bloomington.

Students

Anna Dorman, Dana Paikowsky, and Robin Tholin (all of Harvard Law School).